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Dimensionless numbers for the net present value and the perpetual value of sustainable timber harvests from a monospecific uneven-aged forest

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This paper proposes a simple and direct method to provide reliable approximations of the net present value (NPV) and the perpetual value (PV) of sustainable timber harvests from a monospecific uneven-aged forest based on dimensionless numbers. In addition, two new dimensionless numbers ρ_{NPV} and ρ_{PV} are introduced. These use the NPV or PV derived from the sale of timber throughout a harvest cycle, plus the final stocking value (as numerator), and the fair value of standing timber under IAS 41 (as denominator). They can be interpreted as economic performance indicators for forest management, inspired by the return on assets accounting concept, showing how profitable the forest is, relative to its total value, with sustainability and stability criteria. Those approximations to the variables NPV, PV, ρ_{NPV} and ρ_{PV}, were obtained under conditions of stable equilibrium from a matrix model. In order to exemplify and test the results, the model used data from uneven-aged managed *Pinus nigra* stands, considering three levels of tree diameter growth, six levels of basal area and 33 levels of recruitment, creating a total of 594 planning scenarios. Furthermore, the study revealed the existence of strong linear correlations between those variables and a dimensionless number.

Keywords: Dimensionless Numbers, NPV / PV, Equilibrium, Sustainable Harvesting, Matrix Model, IAS 41

Introduction

By applying conditions of stable equilibrium from a matrix model, a recent study (López & Fullana 2015) found dimensionless numbers $\hat{\lambda}_0$, $\hat{s}$, and $\hat{s}'$ to provide reliable approximations of, respectively, the population growth rate, λ_0, the "sustainable/stable" harvest rate, s, where (eqn. 1):

$$s=\frac{\lambda_0-1}{\lambda_0} \qquad (1)$$

and the proportion of trees that has to remain unharvested in order to retain the stable diameter distribution, s' (where $s' = 1/\lambda_0 = 1-s$), of a forest stand, by means of the following expressions (eqn. 2, eqn. 3, eqn. 4):

$$\hat{\lambda}_0=1+\sqrt[n]{\frac{(2n-1)^2\pi w^2 R\prod_{i=1}^{n-1} p_i}{16G}} \qquad (2)$$

$$\hat{s}=1-\left[1+\sqrt[n]{\frac{(2n-1)^2\pi w^2 R\prod_{i-1}^{n-1} p_i}{16G}}\right]^{-1} =\frac{\hat{\lambda}_0-1}{\hat{\lambda}_0} \qquad (3)$$

$$\hat{s}'=1-\hat{s}=\frac{1}{\hat{\lambda}_0}= =\left[1+\sqrt[n]{\frac{(2n-1)^2\pi w^2 R\prod_{i=1}^{n-1} p_1}{16G}}\right]^{-1} \qquad (4)$$

where R is the global amount of recruitment, G the stand basal area, p_i the transition probabilities between adjacent diameter classes, and n and w, respectively, the number and width of the diameter classes, as detailed below (the use of the circumflex above a variable here denotes a new object that represents an approximation to that variable, as defined in eqn. 2, eqn. 3 and eqn. 4).

Dimensionless numbers are numbers that represent a property of a system, but do not have physical units, such as length, area, mass or time, associated with them. Where such numbers exist for a specific system, they allow conclusions about the behaviour of the system to be extracted, without the need for a more detailed formulation relating all the different variables involved in the problem at hand. Hence, their importance and wide use in many fields, such as mathematics, physics, biology, engineering, and economics (Szirtes 2007).

In financial sciences in general, indicators to quantify financial return have been widely applied, using the internal rate of return (IRR) and the return on assets (ROA – Brealey et al. 2016). The IRR is defined as the discount rate that makes the net present value (NPV) equal to zero, and the ROA is expressed as the quotient between profit and the sum of invested capital. Attempts to derive similar return indicators for forests are discussed in detail in Knoke (2017). Mills & Hoover (1982) calculated the annual change in the market value of standing timber stock and land, divided by the market value of timber and land at the beginning of the year, to obtain an indicator for US timberland. Weber (2002) computed a similar index for German tree species in order to define an indicator for "forest performance", and Hyytiäinen & Penttinen (2008) applied the same perspective in their analyses of forest assets in Finland.

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Tab. 1 - Transition probabilities between diameter classes for each quality. (*n*): number of diameter classes; (*w*): width of the diameter classes (= 6 cm).

n	Quality I	Quality II	Quality III
	9	8	7
(0, 6] → (6, 12]	p_1 = 0.7697	p_1 = 0.5951	p_1 = 0.4564
(6, 12] → (12, 18]	p_2 = 0.8602	p_2 = 0.6824	p_2 = 0.5326
(12, 18] → (18, 24]	p_3 = 0.7913	p_3 = 0.6200	p_3 = 0.4697
(18, 24] → (24, 30]	p_4 = 0.6828	p_4 = 0.5190	p_4 = 0.3692
(24, 30] → (30, 36]	p_5 = 0.5533	p_5 = 0.3971	p_5 = 0.2475
(30, 36] → (36, 42]	p_6 = 0.4106	p_6 = 0.2618	p_6 = 0.1119
(36, 42] → (42, 48]	p_7 = 0.2587	p_7 = 0.1171	-
(42, 48] → (48,→)	p_8 = 0.1000	-	-
$\prod p_k$	0.002104	0.001590	0.001168

In this context, the main aim of this study was to investigate whether there were relationships between those dimensionless numbers (in particular, eqn. 3) and the NPV and PV of sustainable timber harvests from a monospecific uneven-aged forest. In addition, we have introduced two new dimensionless numbers, ρ_{NPV} and ρ_{PV}, using the NPV or PV derived from the sale of timber throughout a harvest cycle, plus the final stocking value (as numerator), and the fair value of the standing timber under IAS 41 (IASB 2000) as denominator. These can be interpreted as economic performance indicators for forest management, inspired by the ROA accounting concept, showing how profitable the forest is, relative to its total value, with sustainability and stability criteria. Moreover, we demonstrated that there are strong linear correlations between those numbers and $\hat{s}$; to that end, it will be used the matrix model applied to uneven-aged managed *Pinus nigra* stands by López et al. (2012, 2013, 2016), which accurately describes the stand dynamics.

Matrix models, introduced by Leslie (1945) and modified by Lefkovitch (1965) by grouping organisms in terms of stage categories rather than age categories, have been applied to almost all the subject areas of forestry (Usher 1969, Caswell 2001, Vanclay 2012, Liang & Picard 2013). In forest management, matrix models have been applied, for example, to evaluate economic outcomes (Buongiorno & Michie 1980, Fortini et al. 2015) and ecological impacts (Van Mantgem & Stephenson 2005, Zhou et al. 2008), or to optimise silviculture in mixed uneven-aged forests to increase the recruitment of browse-sensitive tree species without intervening in the ungulate population (Ficko et al. 2018). A matrix population model works in discrete time, projecting a population from time t to $t + 1$ using a transition matrix (1 here represents the time step or projection interval). It is defined by the finite difference linear system of equations $X(t+1) = AX(t)$, where $X(t)$ and $X(t+1)$ are column vectors containing the number of stems ha^{-1} within each diameter class at time t and t+1, respectively, and A is the transition matrix. In these models, the population growth rate (*i.e.*, the temporal rate of exponential change of the population number of individuals in the long term) is the dominant eigenvalue, λ_o of *A*, and the equilibrium position (stable diameter distribution in this study) is determined by the right eigenvector, W_o, corresponding to λ_o. By asymptotic analysis (long term behaviour), we know that, independent of the initial conditions, when $\lambda_o > 1$, the total number of stems ha^{-1} of the tree population increases exponentially over time (unless harvests are conducted); when $\lambda_o < 1$ the population decays until extinction; and when $\lambda_o = 1$, a stable distribution proportional to W_o is obtained. Gotelli (2001) referred to the special case of the stable distribution when $\lambda_o = 1$ as the "stationary distribution"; we considered the same case in reference to the stable diameter distribution of the stand W_o.

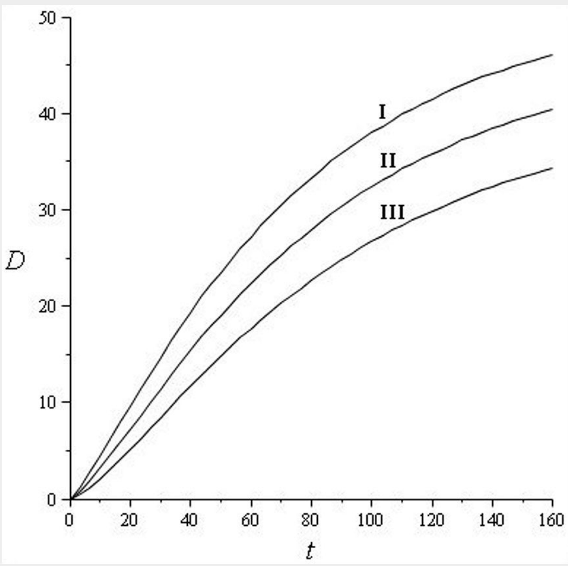


Fig. 1 - Diameter growth models. I: site index 20, $D = 51.68(1 - e^{-0.015259t})^{1.255111}$; II: site index 17, $D = 46.645633(1 - e^{-0.014318t})^{1.337062}$; and III: site index 14, $D = 40.644134(1 - e^{-0.013838t})^{1.456382}$. ($D$): dbh in cm; ($t$): time in years. The coefficient of determination was always greater than 0.999.

Materials and methods

Stand population dynamics model

The matrix model uses the transition probabilities obtained in López et al. (2012, 2013) for uneven-aged managed *Pinus nigra* stands, considering three levels of tree diameter growth (Quality I, faster; Quality II, medium; and Quality III, slower diameter growth – Tab. 1, Fig. 1). The stands are located in the Spanish Iberian System, a mountain range extending about 400 km along the north-eastern edge of the central plateau, concentrating 60% of the area occupied by *Pinus nigra* in Spain (Grande & García Abril 2005). Based on data from those stands, the model simulates outcomes for 594 different forest management planning scenarios: three diameter growth levels (Qualities I-III) × six basal area levels (G = 21 to G = 26 m^2 ha^{-1}) × 33 recruitment levels (R = 200 to R = 840 stems ha^{-1}). Group selection cutting was the silvicultural system used because *Pinus nigra* is not a shade-tolerant species, and the gaps opened in the canopy through single-tree selection cutting would not be large enough to ensure regeneration.

Since the harvesting operations in the study area generally took place every 10 years, this was the time step adopted in the model. Considering this time step, trees were grouped into n diameter classes of equal width, w = 6 cm: (0, 6), (6, 12), (12, 18), …, (30, 36), …, with the last class being (48,→; more than 48 cm) for Quality I, (42,→; more than 42 cm) for Quality II and (36,→; more than 36 cm) for Quality III. Therefore, an individual tree in class k could remain in class k or progress to class k+1 during the projection interval (t, t+1). The number of trees in each class changed in each projection interval, because some were harvested, some remained in the same diameter class and others grew past the boundary to the next diameter class. In such conditions, p_k is the probability that an individual tree in class k at time t (initial) will appear in class k+1 at time t+1 (final time of projection); the recruitment coefficient, r_k, is the number of offspring (stems ha^{-1}) living at time t+1 produced in the interval of projection (t, t+1) by an average tree in class k at time t (similar to many standard size classified matrix models, $r_1 = 0$ – Caswell 2001); $h_k(t)$ defines the proportion of harvested trees in class k, natural mortality included; $x_k(t)$ and $x_k(t+1)$ describe the stem densities in class k at the initial and final times of projection. Finally, $R = r_2 x_2 + r_3 x_3 + \dots + r_n x_n$ denotes the global amount of recruitment at each time step, defined as the total number of trees entering into the first diameter class from t to t+1, generally above a certain threshold diameter. Notice that $0 < p_k < 1$ for $k = 1, 2, \dots, n$-1; $r_k \geq 0$ for $k = 2, 3, \dots$, n, with $R = 0$, if and only if $r_k = 0$ for all k; and $x_k > 0$ for $k = 1, 2, \dots, n$.

By analysing the dynamics of the projections, we found that the model is described accurately by the matrix model (eqn. 5):

$$X(t+1)=A[I-H(t)]X(t) \tag{5}$$

where (eqn. 6):

$$A=\begin{bmatrix} 1-p_1 & r_2 & r_3 & \cdots & r_{n-1} & r_n \\ p_1 & 1-p_2 & 0 & \cdots & 0 & 0 \\ 0 & p_2 & 1-p_3 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1-p_{n-1} & 0 \\ 0 & 0 & 0 & \cdots & p_{n-1} & 1 \end{bmatrix} \tag{6}$$

I is the identity matrix of order n, $H(t) = diag[h_1(t), h_2(t), \dots, h_n(t)]$ is a diagonal matrix with the harvest rates $h_k(t)$, including natural mortalities, and $X(t)$ and $X(t+1)$ are column vectors indicating the stem densities at the initial and final times of projection, respectively.

Sustainable/stable harvesting strategy and stable diameter distribution

The "sustainable/stable" harvesting strategy is aimed at reaching the stable diameter distribution of the stand for each harvest, yielding the harvest rate, s, shown in eqn. 1 for all the diameter classes and time steps; the population growth rate, λ_o, is the dominant eigenvalue of the transition matrix without harvests (*i.e.*, matrix A), and s also includes the natural mortalities. This is a sustainable harvesting strategy too (in the sense of population persistence over time) because s corresponds to the case in which the right eigenvector associated with the dominant eigenvalue equal to 1 of the transition matrix with harvests, *i.e.*, matrix $A(I-H)$, is W_o.

On the other hand, the equilibrium is defined by the stable diameter distribution of the stand W_o, which is simultaneously the right eigenvector of the matrices A and $A(I-H)$ corresponding, respectively, to the dominant eigenvalues λ_o and 1. Thus the components of the stable diameter distribution of the stand $N_1, N_2, \dots, N_n$ (stems ha^{-1}) were obtained by solving the linear system $AW_o = \lambda_o W_o$, yielding (proportional to) the vector $W_o = (N_1, N_2, N_3, \dots, N_{n-1}, N_n)$, where (eqn. 7):

$$N_i=(\lambda_0-1)\prod_{k=2}^{n-1}(\lambda_0-1+p_k),$$
$$N_i=(\lambda_0-1)\left(\prod_{k=1}^{i-1}p_k\right)\left(\prod_{k=i+1}^{n-1}(\lambda_0-1+p_k)\right) \tag{7}$$

for 1< i < n-1, and (eqn. 8):

$$N_{n-1}=(\lambda_0-1)\prod_{k=1}^{n-2}p_k,$$
$$N_n=\prod_{k=1}^{n-1}p_k \tag{8}$$

NPV function

Considering that the main aim for the stands in the study area is the production of timber, the objective function is the NPV of all management operations over a planning horizon of T years, discounted to the beginning of the period, that is (eqn. 9):

$$NPV=\sum_{k=1}^{n}\sum_{t=0}^{T-10}x_k(t)h_k(t)v_k(t)\frac{1}{(1+i)^t}+\sum_{k=1}^{N}x_k(T)v_k(T)\frac{1}{(1+i)^T} \tag{9}$$

where $v_k(t)$ is the stumpage value that corresponds to class k at year t (€ stem^{-1}) and i is the discount rate (López et al. 2013, 2016). The first summation represents the income derived from the sale of timber, and the second is the final stocking value. Harvesting costs are not considered because it is assumed that harvesting operations are not paid for the owner, but for the buyer of the stumpage. The costs for the owner of the land are the opportunity cost of invested capital and the opportunity cost of land. None of them are explicit costs, but they are incorporated into the NPV function. It is well known that the NPV is a justifiable management objective for a single economic goal (Siegel et al. 1995).

The corresponding function of that management regime maintained in perpetuity is given by (eqn. 10):

$$PV=\sum_{k=1}^{n}\sum_{t=0}^{\infty}x_k(t)h_k(t)v_k(t)\frac{1}{(1+i)^t} \tag{10}$$

In the case of the equilibrium defined by the "sustainable/stable" harvesting strategy, since $h_k(t) = s$, $x_k(t) = N_k$, for $k = 1, 2, \dots, n$ and $t = 0, 10, 20, \dots$, the above equations become (eqn. 11, eqn. 12):

$$NPV=\sum_{k=1}^{n}\sum_{t=0}^{T-10}N_k s v_k(t)\frac{1}{(1+i)^t}+\sum_{k=1}^{n}N_k v_k(t)\frac{1}{(1+i)^T} \tag{11}$$

$$PV=\sum_{k=1}^{n}\sum_{t=0}^{\infty}N_k s v_k(t)\frac{1}{(1+i)^t} \tag{12}$$

Tab. 2 - Stumpage prices (€ m^{-3}) of *Pinus nigra* in Spain according to diameter class and industrial destination.

Products	Diameter classes (cm)		
	<20	20-40	>40
Particle	6	1.2	0.6
Poles	0	11.85	0
Sawlog	0	9.8	11.76
High-quality sawlog	0	0	9.96
Total average price	6	22.85	22.32

Since the stumpage prices of *Pinus nigra* in Spain remain fairly stable through time (MAGRAMA 2012), simplifying eqn. 11 and eqn. 12 we obtain (eqn. 13, eqn. 14):

$$NPV=s\sum_{k=1}^{n}N_k v_k\sum_{t=0}^{T-10}\frac{1}{(1+i)^t}+\sum_{k=1}^{n}N_k v_k\frac{1}{(1+i)^T}=\left[s\frac{(1+i)^{-T}-1}{(1+i)^{-10}-1}+\frac{1}{(1+i)^T}\right]\sum_{k=1}^{n}N_k v_k \tag{13}$$

$$PV=s\sum_{k=1}^{n}N_k v_k\sum_{t=0}^{\infty}\frac{1}{(1+i)^t}=s\frac{1}{1-(1+i)^{-10}}\sum_{k=1}^{n}N_k v_k \tag{14}$$

where $\sum N_k v_k$ represents the fair value of standing timber under IAS 41 (IASB 2000). By dividing eqn. 13 and eqn. 14 by this value, we obtain (eqn. 15, eqn. 16):

$$\rho_{NPV}=\frac{NPV}{\sum_{k=1}^{n}N_k v_k}=s\frac{(1+i)^{-T}-1}{(1+i)^{-10}-1}+\frac{1}{(1+i)^T} \tag{15}$$

$$\rho_{PV}=\frac{PV}{\sum_{k=1}^{n}N_k v_k}=s\frac{1}{1-(1+i)^{-10}} \tag{16}$$

which means that, for previously defined values of i and T, there is a positive linear correlation between the NPV weighted by the fair value of standing timber, $\sum N_k v_k$, that is ρ_{NPV}, and s (resp. between the PV weighted by $\sum N_k v_k$, that is ρ_{PV}, and s). Note that ρ_{NPV} and ρ_{PV} are dimensionless numbers. Furthermore, given that there is a strong positive linear correlation between s and $\hat{s}$, (López & Fullana 2015), it is expected that, for previously defined values of i and T, there must be a strong positive linear correlation between these variables ρ_{NPV} and ρ_{PV}, and $\hat{s}$, as we demonstrate below.

Stumpage value model

The stumpage value model applied in this study comes from López et al. (2013, 2016). In Tab. 2, we provide the stumpage values we used to obtain the economic data (Montero et al. 1992, Trasobares & Pukkala 2004). The regression models adjusted to these data appear in Fig. 2. The stumpage prices of *Pinus nigra* in Spain remained fairly stable between 1994 and 2011 (MAGRAMA 2012).

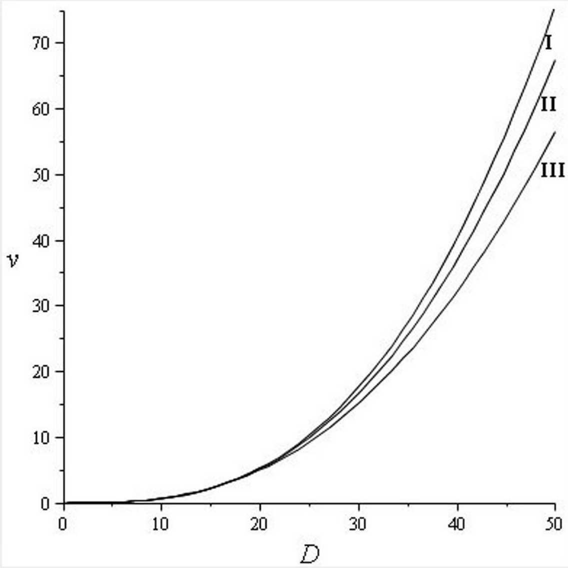


Fig. 2 - Stumpage value models: Quality I: $v = D^{3.186471} \exp(-7.704952 - 8.678687 \cdot 10^{-3} D)$; Quality II: $v = D^{3.114196} \exp(-7.476506 - 9.903125 \cdot 10^{-3} D)$; and Quality III: $v = D^{2.987053} \exp(-7.110977 - 1.078752 \cdot 10^{-2} D)$; stumpage values ($v$) in € stem^{-1} and diameter (D) in cm. The coefficient of determination was always greater than 0.9999.

Tab. 3 - Numerical values of s and $\hat{s}$ (as defined in eqn. 4) for Qualities I, II and III.

Quality	G (m^2 ha^{-1})	Variable	R = 200 stem ha^{-1}	R = 520 stem ha^{-1}	R = 840 stem ha^{-1}
Q I	22	s	0.233770	0.323259	0.372742
		$\hat{s}$	0.350669	0.375210	0.387782
	24	s	0.226305	0.314580	0.363570
		$\hat{s}$	0.348471	0.372946	0.385489
	26	s	0.219546	0.306683	0.355205
		$\hat{s}$	0.346454	0.370868	0.383384
Q II	22	s	0.207665	0.292049	0.339704
		$\hat{s}$	0.318779	0.345258	0.358932
	24	s	0.200729	0.283764	0.330816
		$\hat{s}$	0.316422	0.342804	0.356433
	26	s	0.194462	0.276245	0.322732
		$\hat{s}$	0.314262	0.340553	0.354142
Q III	22	s	0.180733	0.259707	0.305332
		$\hat{s}$	0.278309	0.306537	0.321290
	24	s	0.174348	0.251850	0.296767
		$\hat{s}$	0.275820	0.303901	0.318585
	26	s	0.168593	0.244738	0.288999
		$\hat{s}$	0.273542	0.301487	0.316108

The last row in Tab. 2 shows the average price per cubic metre for trees belonging to the different diameter classes. This price was obtained by adding up the four rows above in Tab. 2 that show the contributions to the final value of a tree for the four industrial destinations according to the percentage of wood of each diameter class going into each product type. The prices of particle board, poles, sawlogs, and high-quality sawlogs were 6.0, 39.5, 19.6 and 33.2 € m^{-3}, respectively. It is necessary to point out that the demand for timber poles in Spain is very high, and therefore they reach a higher price than larger-sized sawlogs.

Results

By substituting transition probabilities p_i, recruitment R, basal area G, together with w and n into the eqn. 3, we obtained $\hat{s}$, as summarised in Tab. 3, for selected scenarios.

On the other hand, by applying eqn. 7 and eqn. 8, we obtained the components N_k of the stable diameter distributions W_o of the stand, as summarized in Tab. 4 for some scenarios. The λ_o values were obtained for each scenario under conditions of stable equilibrium from eqn. 5, using the transition matrix without harvests, and the s values came from eqn. 1.

On the basis of the above data, by substituting N_k, s, $\hat{s}$, i, T and v_k into eqn. 13 to eqn. 16, we obtained NPV, PV, ρ_{NPV} and ρ_{PV}, as summarised in Tab. 5 and Tab. 6, for i = 3%, T = 90 years and several scenarios. Additionally, Fig. 3 and Fig. 4 depict the points ($\hat{s}$, NPV) for T = 90, ($\hat{s}$, PV), ($\hat{s}$, ρ_{NPV}) for T = 90, and ($\hat{s}$, ρ_{PV}), respectively (198 points for each case).

The discount rates applied fell between 2% and 4%, as is typical in forest management studies, and the time horizon used to calculate the NPV ranged between 50 and 130 years. The influence of small variations in the discount rate i, or the time horizon T, on the results was minimal because they did not change the behaviour of the expressions shown in Materials and Methods (chapter "NPV functions").

Regarding the regressions, we obtained the following results:

- As already mentioned, for previously defined values of i and T, the dimensionless numbers ρ_{NPV} and $\hat{s}$ have a strong positive linear correlation (see Fig. 4). For i = 3%, in particular, the total proportion of variance in ρ_{NPV} explained by $\hat{s}$ was r^2 = 0.99974 for Quality I, r^2 = 0.99965 for Quality II, and r^2 = 0.99956 for Quality III. It is important to note that such r^2 values do not depend on the planning horizon T.
- Similarly, for each discount rate i, the dimensionless numbers ρ_{PV} and $\hat{s}$ have a strong positive linear correlation, with the same r^2 values as described above (Fig. 4).

Tab. 4 - Stable diameter distributions (W_o). Components in stem ha^{-1}, as defined in eqn. 7 and eqn. 8.

Quality	G (m^2 ha^{-1})	R = 200 stem ha^{-1}	R = 840 stem ha^{-1}
Q I	22	[186.1, 122.9, 96.4, 77.2, 61.4, 47.5, 34.6, 22.1, 7.2]	[615.9, 325.9, 202.3, 125.4, 74.6, 41.1, 19.8, 7.4, 1.2]
	24	[188.3, 125.7, 99.8, 81.0, 65.4, 51.4, 38.3, 25.3, 8.6]	[626.4, 336.8, 212.6, 134.2, 81.5, 45.9, 22.7, 8.8, 1.5]
	26	[190.3, 128.3, 102.9, 84.5, 69.1, 55.3, 42.0, 28.5, 10.1]	[636.1, 347.0, 222.4, 142.6, 88.2, 50.8, 25.7, 10.2, 1.9]
Q II	22	[233.3, 147.0, 113.7, 90.3, 71.1, 53.9, 37.2, 16.6]	[757.0, 376.4, 226.4, 135.8, 77.3, 39.6, 16.4, 3.7]
	24	[236.3, 150.7, 118.0, 95.0, 76.1, 58.9, 41.9, 19.5]	[771.0, 389.9, 238.8, 146.1, 85.0, 44.7, 19.1, 4.5]
	26	[239.1, 154.0, 122.0, 99.5, 80.9, 63.8, 46.6, 22.6]	[783.9, 402.5, 250.5, 156.0, 92.7, 49.8, 22.0, 5.4]
Q III	22	[295.4, 179.0, 138.1, 110.0, 86.7, 64.6, 32.8]	[937.6, 440.2, 257.8, 149.7, 80.5, 36.1, 9.2]
	24	[299.6, 183.8, 143.8, 116.4, 93.7, 71.8, 38.0]	[956.3, 457.2, 273.0, 162.1, 89.4, 41.4, 11.0]
	26	[303.4, 188.3, 149.1, 122.5, 100.4, 79.0, 43.6]	[973.6, 473.1, 287.6, 174.1, 98.3, 46.9, 12.9]

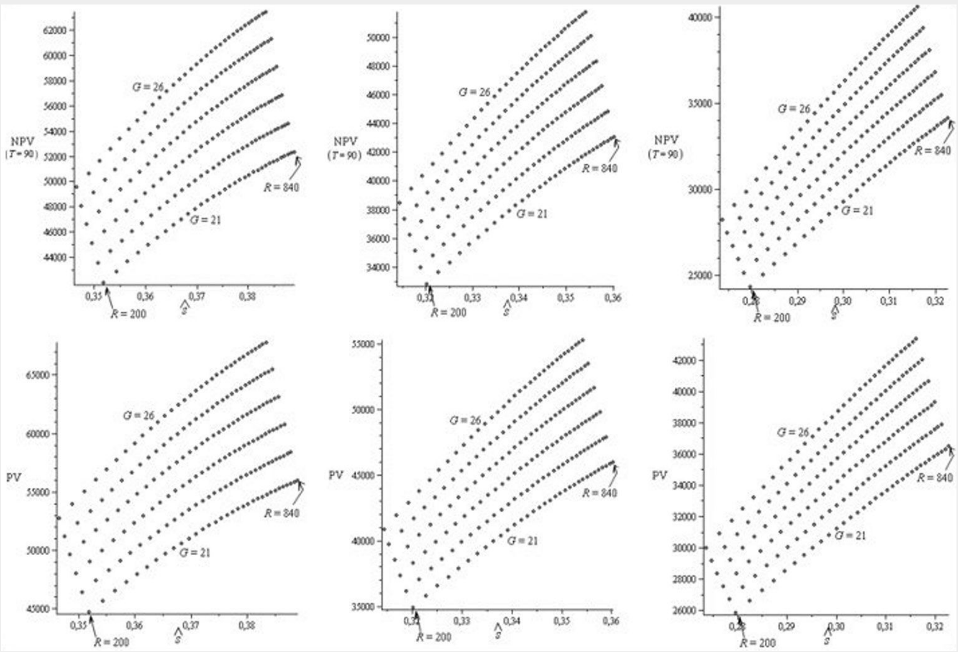
Tab. 5 - NPV (for *T* = 90 years) and PV values (€ ha^{-1}) for the "sustainable/stable" harvesting strategies for *i* = 3% and Qualities I, II and III.

Quality	G (m^2 ha^{-1})	Param	*R* = 200 stem ha^{-1}	*R* = 520 stem ha^{-1}	*R* = 840 stem ha^{-1}
Q I	22	NPV	5319.159	6191.831	6500.993
		PV	5284.168	6283.379	6646.680
	24	NPV	5701.675	6682.311	7038.929
		PV	5649.995	6770.617	7187.771
	26	NPV	6074.394	7163.932	7569.088
		PV	6004.851	7247.846	7720.004
Q II	22	NPV	4185.287	5021.953	5359.973
		PV	4118.386	5065.793	5454.054
	24	NPV	4474.700	5404.061	5785.948
		PV	4390.309	5441.416	5879.039
	26	NPV	4756.044	5778.243	6204.543
		PV	4653.231	5808.146	6295.701
Q III	22	NPV	3131.227	3909.704	4264.179
		PV	3042.728	3913.709	4313.000
	24	NPV	3336.493	4191.150	4584.694
		PV	3230.809	4186.435	4629.266
	26	NPV	3535.449	4465.779	4898.460
		PV	3411.887	4451.574	4938.000

Tab. 6 - ρ_{NPV} (for *T* = 90 years) and ρ_{PV} dimensionless numbers for the "sustainable/stable" harvesting strategies for *i* = 3% and Qualities I, II and III.

Quality	G (m^2 ha^{-1})	Param	*R* = 200 stem ha^{-1}	*R* = 520 stem ha^{-1}	*R* = 840 stem ha^{-1}
Q I	22	ρ_{NPV}	0.9195	1.2448	1.4246
		ρ_{PV}	0.9135	1.2632	1.4566
	24	ρ_{NPV}	0.8924	1.2132	1.3913
		ρ_{PV}	0.8843	1.2293	1.4207
	26	ρ_{NPV}	0.8679	1.1845	1.3609
		ρ_{PV}	0.8579	1.1984	1.3880
Q II	22	ρ_{NPV}	0.8247	1.1314	1.3046
		ρ_{PV}	0.8115	1.1412	1.3275
	24	ρ_{NPV}	0.7995	1.1012	1.2723
		ρ_{PV}	0.7844	1.1089	1.2927
	26	ρ_{NPV}	0.7767	1.0739	1.2429
		ρ_{PV}	0.7599	1.0795	1.2611
Q III	22	ρ_{NPV}	0.7268	1.0138	1.1796
		ρ_{PV}	0.7062	1.0149	1.1931
	24	ρ_{NPV}	0.7036	0.9853	1.1485
		ρ_{PV}	0.6813	0.9841	1.1597
	26	ρ_{NPV}	0.6827	0.9594	1.1203
		ρ_{PV}	0.6588	0.9564	1.1293

Fig. 3 - First row: set of points ($\hat{s}$, NPV) for *T* = 90; second row: set of points ($\hat{s}$, PV). Left: Quality I; middle: Quality II; right: Quality III (198 total points for each case, 33 points for each basal area level, six points for each recruitment level, and *i* = 3%).



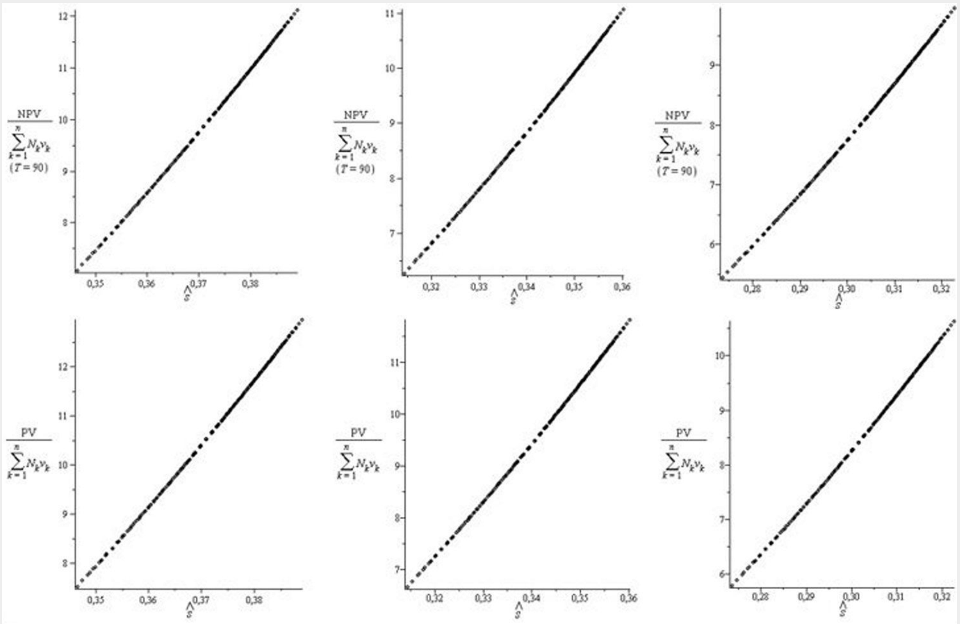


Fig. 4 - First row: set of points ($\hat{s}$, ρ_{NPV}) for T = 90; second row: set of points ($\hat{s}$, ρ_{PV}). Left: Quality I; middle: Quality II; right: Quality III (198 points for each case, and i = 3%).

- For previously defined values of basal area (between G = 21 m^2 ha^{-1} and G = 26 m^2 ha^{-1}), i and T, the variables NPV and $\hat{s}$ have a strong positive linear correlation (Fig. 3). The same result holds for the variables PV and $\hat{s}$. The best approximations always occurred for the lowest level of diameter growth and the highest basal area scenarios, with r^2 being very close to 1 in all cases.
- For previously defined values of recruitment level (between R = 200 and R = 840 stems ha^{-1}), i and T, the variables NPV and $\hat{s}$ have a strong negative linear correlation (Fig. 3). The same result holds for the variables PV and $\hat{s}$. The best approximations always occurred for the lowest level of diameter growth and the lowest recruitment scenarios, with r^2 being very close to 1 in all cases.

Discussion and conclusions

This paper proposes a simple and direct method based on dimensionless numbers, obtained under conditions of stable equilibrium from a matrix model, to provide reliable approximations of the NPV, PV, ρ_{NPV} and ρ_{PV} of a monospecific uneven-aged stand. In fact, these variables were related to $\hat{s}$ (see eqn. 3), which is a dimensionless number based on the following variables: (a) the transition probabilities p_k, which can be calculated from counts of observed transitions between adjacent diameter classes from t to t+1; (b) the basal area of the stand G, and the global amount of recruitment R. Although *Pinus nigra* regenerates well in the study area under its own canopy (Retana et al. 2002, Tiscar 2007), there is a critical basal area that allows sufficient light transmittance for regeneration. This maximum value is around 20-30 m^2 ha^{-1} (Schütz 1989, Serrada et al. 1994, Monge Reyes 1997). On the other hand, basal area clearly below 20 m^2 ha^{-1} (before harvesting) could compromise natural regeneration (Serrada et al. 1994). Consequently, in addition to three levels of diameter growth (Qualities I-III), we considered six levels of stand basal area from G = 21 to G = 26 m^2 ha^{-1}, and 33 levels of global recruitment, from R = 200 stems ha^{-1} (scarce recruitment) to R = 840 stems ha^{-1} (abundant recruitment), creating a total of 594 planning scenarios. The corresponding problems simulated a wide variety of typical management cases in the study area; and (c) the number n and width w of the diameter classes. As shown in López & Fullana (2015), these scaling factors n and w could have a limited influence on $\hat{s}$ and therefore on the NPV, PV, ρ_{NPV} and ρ_{PV} values.

The ρ_{NPV} and ρ_{PV} dimensionless numbers should not be interpreted as a kind of IRR, but as economic performance indicators for forest management inspired by the ROA accounting concept, using the NPV or PV derived from the sale of timber throughout a harvest cycle, plus the final stocking value (as numerator), and the fair value of standing timber (as denominator), showing how profitable the forest is relative to its total value, with sustainability and stability criteria. Application of the NPV (or PV) implies the introduction of a discount rate exogenously defined. In this regard, we applied the same criteria as in the International Financial Reporting Standards (IFRS), in particular IAS 41 (IASB 2000), to estimate that exogenous discount rate. In fact, IAS 41 (IASB 2000), in applying the expectation value approach to forest valuation, when referring to the process of deriving the present value of expected net cash flows for a forest stand, prescribed the use of a determined discount rate in each case, recommending that assumptions about these rates should be internally consistent with the underlying economic factors of the sector. By using a discount rate in accordance with international accounting standards, the possibility of manipulation should be avoided or limited. Thus, to consider the incorporation of eventual risks, in this study we initially chose a wide range of discount rates from 2% to 4%, typical of forest management scenarios (Hyytiäinen & Penttinen 2008, HM-Treasury 2013, Knoke 2017). Note that (eqn. 17):

$$\lim_{T \to +\infty} \rho_{NPV} = \rho_{PV} \tag{17}$$

It is not surprising therefore that the highest ρ_{NPV} and ρ_{PV} values were obtained for Quality I, followed by Quality II and Quality III (see Tab. 6 and Fig. 4). In fact, since there is a strong positive linear correlation between these numbers and $\hat{s}$, and considering that the core of this dimensionless number $\hat{s}$ is given by the quotient (eqn. 18):

$$\frac{R \prod_{i=1}^{n-1} p_i}{G} \tag{18}$$

under the same conditions for R and G, the faster the flow of the stems through the diameter classes (*i.e.*, the higher product of the transition probabilities), the greater $\hat{s}$ and consequently the greater the ρ_{NPV} and ρ_{PV} values. Likewise, within each site quality (where the product of the transition probabilities may be assumed to be constant), the key factor underlying the ρ_{NPV} and ρ_{PV} equations is the quotient R/G, which means that the greater R/G, the more profitable the forest will be (i.e., the G = 21 m^2 ha^{-1} with R = 840 stems ha^{-1} is the most favourable scenario in this context). Two arrows in Fig. 4 point in the directions of the higher and lower R/G values.

Furthermore, the strong positive linear correlation between ρ_{NPV} and ρ_{PV} and $\hat{s}$ means that, stratifying the forest into uniform areas based upon tree diameter growths, the same linear regression line can be used in each stratum to estimate the ρ_{NPV} and ρ_{PV} dimensionless numbers. The same result holds for the variables NPV or PV and $\hat{s}$ for each basal area or recruitment levels, although, in the latter case, the corresponding strong linear correlation is negative.

Since the model rests on sustainability and stability criteria, it is important to investigate the sensitivity of the results to deviations from the equilibrium (stable diameter distribution); in particular, whether the strong linear relationship between the dimensionless numbers, ρ_{NPV} and ρ_{PV}, and $\hat{s}$ is sensitive to changes in the equilibrium position. In this regard, there have been two recent studies involving *Pinus nigra* stands in the study area (López et al. 2013, 2016), both maximising the same NPV function, constrained by the same matrix model to describe the population dynamics, and with the same initial condition (the stable diameter distribution), as used in this study. Additionally, in the second one the final condition was assumed as the stable diameter distribution of the stand, and a wide variety of typical management scenarios in the study area, obtained by combining three rates of diameter growth (Qualities I-III), three levels of basal area (G = 22, 24 and 26 m^2 ha^{-1}), and three levels of recruitment (*R* = 200, 520 and 840 stems ha^{-1}), were considered. The first one (hereafter S1) showed that the economically optimal harvesting strategies underlying the solutions for that model changed the stand towards distributions substantially deviated from the stable diameter distribution; and the second (hereafter S2) revealed that the stand diameter distribution did not deviate substantially from the equilibrium position over time. The NPV values obtained for the optimal harvesting strategies from S2 were always below those of the optimal harvesting strategies from S1 (where no restrictions were introduced for the final state), and above the NPV values obtained under sustainability and stability criteria.

In each case, the ρ_{NPV} and ρ_{PV} dimensionless numbers were computed, and a regression analysis was conducted, using the same $\hat{s}$ values as in this study. The results showed that, in both S1 and S2 strategies the strong linear relationship between the dimensionless numbers ρ_{NPV} and ρ_{PV}, and $\hat{s}$ was not sensitive to changes in the equilibrium position, with r^2 values being very close to 1 in all cases. The lowest r^2 values always occurred for the lowest level of diameter growth, with the minimum value of r^2 being 0.9986 for the strategy S1.

In conclusion, this study showed that, under sustainability and stability criteria, there is a strong influence of the dimensionless number $\hat{s}$ on the NPV and PV of the forest harvesting strategies. Additionally, ρ_{NPV} and ρ_{PV} might be interpreted as economic performance indicators for forest management, inspired by the ROA accounting concept, showing how profitable the forest is relative to its total value, with sustainability and stability criteria. Those dimensionless numbers would also allow the extraction of conclusions about the behaviour of NPV of the forest harvesting, capturing information of all the global variables involved in the problem (*e.g.*, recruitment, basal area and transition probabilities between diameter classes). Moreover, the study showed that there are strong linear correlations between the variables, NPV, PV, ρ_{NPV} , ρ_{PV}, and $\hat{s}$.

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References

Brealey RA, Myers SC, Allen F (2016). Principles of corporate finance (12th edn). McGraw-Hill/Irwin, New York, USA, pp. 976.

Buongiorno J, Michie BR (1980). A matrix model of uneven-aged forest management. Forest Science 26 (4): 609-625. [online] URL: http://academic.oup.com/forestscience/article-abstract/26/4/609/4656453

Caswell H (2001). Matrix population models: construction, analysis, and interpretation (2nd edn). Sinauer Associates Inc., Sunderland, MA, USA, pp. 710.

Ficko A, Roessiger J, Bončina A (2018). Optimizing silviculture in mixed uneven-aged forests to increase the recruitment of browse-sensitive tree species without intervening in ungulate population. iForest - Biogeosciences and Forestry 11: 227-236. - doi: 10.3832/ifor2567-011

Fortini LB, Cropper WP, Zarin DJ (2015). Modeling the complex impacts of timber harvests to find optimal management regimes for Amazon tidal floodplain forests. PLoS ONE 10 (8): e0136740. - doi: 10.1371/journal.pone.0136740

Gotelli NJ (2001). A primer of ecology (3rd edn). Sinauer Associates, Sunderland, MA, USA, pp. 290.

Grande MA, García Abril AE (2005). Los Pinares de *Pinus nigra* Arn. en España: ecología, uso y gestión. [The forests of *Pinus nigra* Arn. in Spain: ecology, use and management]. Fundación Conde del Valle de Salazar (FUCOVASA), Madrid, Spain, 2005, pp. 699. [in Spanish]

HM-Treasury (2013). The green book: appraisal and evaluation in central government. HM Treasury, London, UK, pp. 132. [online] URL: http://www.gov.uk/government/publications/the-green-book-appraisal-and-evaluation-in-central-governent

Hyytiäinen K, Penttinen MJ (2008). Applying portfolio optimisation to the harvesting decisions of non-industrial private forest owners. Forest Policy and Economics 10 (3): 151-160. - doi: 10.1016/j.forpol.2007.07.002

IASB (2000). International accounting standard 41 (IAS 41 Agriculture, december 2000). International Accounting Standards Board (IASB), International Financial Reporting Standards Foundation, London, UK, pp. A1293-A1305.

Knoke T (2017). Economics of mixed forests. In: "Mixed-Species Forests: Ecology and Management" (Pretzsch H, Forrester DI, Bauhus J eds). Springer-Verlag GmbH, Berlin, Germany, pp. 545-579. - doi: 10.1007/978-3-662-54553-9_11

Lefkovitch LP (1965). The study of population growth in organisms grouped by stages. Biometrics 21: 1-18. - doi: 10.2307/2528348

Leslie PH (1945). On the use of matrices in certain population mathematics. Biometrika 33: 183-212. - doi: 10.1093/biomet/33.3.183

Liang J, Picard N (2013). Matrix model of forest dynamics: an overview and outlook. Forest Science 59 (3): 359-378. - doi: 10.5849/forsci.11-123

López I, Fullana C (2015). Dimensionless numbers for the sustainable harvesting of a monospecific uneven-aged forest. Canadian Journal of Forest Research 45: 1535-1545. - doi: 10.1139/cjfr-2015-0140

López I, Ortuño SF, García Robredo F, Fullana C (2012). Is De Liocourt's distribution stable? Forest Science 58 (1): 34-46. - doi: 10.5849/forsci.10-015

López I, Ortuño SF, García Robredo F, Fullana C (2013). Are the economically optimal harvesting strategies of uneven-aged *Pinus nigra* stands always sustainable and stabilizing? Forests 4: 830-848. - doi: 10.3390/f4040830

López I, Ortuño S, García Robredo F, Fullana C (2016). Optimizing the management of uneven-aged *Pinus nigra* stands between two stable positions. iForest - Biogeosciences and Forestry 9: 599-607. - doi: 10.3832/ifor1298-008

MAGRAMA (2012). Boletín mensual de estadística 1994-2011 [Monthly Statistical Bulletin 1994-2011]. Technical General Secretary, Spanish Ministry of Agriculture, Food and Environment, Madrid, Spain, pp. 48. [in Spanish]

Mills WL, Hoover W (1982). Investment in forest land: aspects of risk and diversification. Land Economics 58 (1): 33-51. - doi: 10.2307/3146075

Monge Reyes A (1997). Selvicultura y regeneración de *Pinus nigra* en las sierras Béticas [Forestry and regeneration of *Pinus nigra* in the Baetic mountain-range]. Doctoral thesis, ETSI de Montes de la UPM, Madrid, Spain, pp. 511. [in Spanish]

Montero G, Rojo A, Alía R (1992). Determinación del turno de *Pinus sylvestris* L. en el Sistema Central [Determination of rotation age for *Pinus sylvestris* L. in the Central System]. Revista Montes 29: 42-47. [in Spanish]

Retana J, Espelta JM, Habrouk A, Ordóñez JL, Solà-Morales F (2002). Regeneration patterns of three Mediterranean pines and forest changes after a large wildfire in north-eastern Spain. Ecoscience 9: 89-97. - doi: 10.1080/11956860.2002.11682694

Schütz JP (1989). Le régime du Jardinage. [The gardening regime]. Chaire de sylviculture, ETH Zürich, Switzerland, pp. 110. [in French]

Serrada R, Domínguez Lerena S, Sánchez Resco MI, Ruiz Ortiz J (1994). El problema de la regeneración natural de *Pinus nigra* Arn. [The problem of natural regeneration of *Pinus nigra* Arn.]. Revista Montes 36: 52-57. [in Spanish]

Siegel W, Hoover W, Haney H, Liu K (1995). Forest owners' guide to the federal income tax. Agriculture Handbook No. 708, USDA Forest Service, Washington, DC, USA, pp. 138.

Szirtes T (2007). Applied dimensional analysis and modeling (2nd edn). Elsevier Butterworth-Heinemann, Burlington, MA, USA, pp. 856.

Tiscar PA (2007). Dinámica de regeneración de *Pinus nigra* subs. *salzmannii* al sur de su área de distribución: etapas, procesos y factores implicados. [Regeneration dynamics of *Pinus nigra* subs. *salzmannii* at the southeast of its distribution area: stages, processes and factors involved]. Investigación Agraria: Sistemas y Recursos Forestales 16 (2): 124-135. [in Spanish] - doi: 10.5424/srf/2007162-01003

Trasobares A, Pukkala T (2004). Optimising the management of uneven-aged *Pinus sylvestris* L. and *Pinus nigra* Arn. mixed stands in Catalonia, north-east Spain. Annals of Forest Science 61: 747-758. - doi: 10.1051/forest:2004071

Usher MB (1969). A matrix model for forest management. Biometrics 25 (2): 309-315. - doi: 10.2307/2528791

Vanclay JK (2012). Modelling continuous cover forests. In: "Continuous Cover Forestry" (Pukkala T, Von Gadow K eds). Managing Forest Ecosystems, vol. 23, Springer, Dordrecht, Netherlands, pp. 229-241. - doi: 10.1007/978-94-007-2202-6_7

Van Mantgem PJ, Stephenson NT (2005). The accuracy of matrix population model projections for coniferous trees in the Sierra Nevada, California. Journal of Ecology 93: 737-747. - doi: 10.1111/j.1365-2745.2005.01007.x

Weber MW (2002). Portefeuille- und real-options preist- theorie und forstliche entscheidungen [Portfolio and real option price- theory and forestry decisions]. Sauerländer's Verlag, Frankfurt, Germany, pp. 197. [in German]

Zhou M, Liang J, Buongiorno J (2008). Adaptive versus fixed policies for economic or ecological objectives in forest management. Forest Ecology and Management 254 (2): 178-187. - doi: 10.1016/j.foreco.2007.07.035