Modelling diameter distribution of *Tetraclinis articulata* in Tunisia using normal and Weibull distributions with parameters depending on stand variables

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The objective of this study was to evaluate the effectiveness of both Normal and two-parameter Weibull distributions in describing diameter distribution of *Tetraclinis articulata* stands in north-east Tunisia. The parameters of the Weibull function were estimated using the moments method and maximum likelihood approaches. The data used in this study came from temporary plots. The three diameter distribution models were compared firstly by estimating the parameters of the distribution directly from individual tree measurements taken in each plot (parameter estimation method), and secondly by predicting the same parameters from stand variables (parameter prediction method). The comparison was based on bias, mean absolute error, mean square error and the Reynolds' index error (as a percentage). On the basis of the parameter estimation method, the Normal distribution gave slightly better results, whereas the Weibull distribution with the maximum likelihood approach gave the best results for the parameter prediction method. Hence, in the latter case, the Weibull distribution with the maximum likelihood approach appears to be the most suitable to estimate the parameters for reducing the different comparison criteria for the distribution of trees by diameter class in *Tetraclinis articulata* forests in Tunisia.

Keywords: Diameter Class Model, Normal Distribution, Weibull Distribution, Maximum Likelihood Approach, Moments Method, *Tetraclinis articulata*

Introduction

Thuya (*Tetraclinis articulata* [Vahl.] Mast.) is a semiarid Mediterranean forest species which extends discontinuously from North Africa to south-western Europe. The natural character and the limited geographic distribution area of the species make it of particular ecological interest in terms of biodiversity conservation. Thuya forests provide highly valuable services such as soil erosion control, biodiversity conservation and CO_2 fixation, and are suitable for afforestation programs in arid or semiarid environments, where few species are able to grow (Tenbergen et al. 1995), and areas with severely eroded soils (Esteve-Selma et al. 2010).

In addition, these stands are of potential economic and social interest for the rural populations, not only in Tunisia where they cover 33,000 ha (DGF 1995), but also in Algeria and Morocco, covering more than 1,000,000 ha in Maghreb (Charco 1999). Thuya timber is widely used in construction, handicraft and cabinetmaking due to its hardness, veneer, durability and fragrance. A secondary traditional use of this species is resin production to obtain sandarac gum. However, overexploitation for timber, irrational resin tapping, grazing pressure and uncontrolled fires have resulted in highly degraded forests (Charco 1999, El-Mouridi et al. 2011). The extinction of this species has only been avoided by its resprouting capacity. The current degraded state has left these forests devoid of large trees, so the main production presently comprises small pieces of wood for fuel and fencing (DREF 2002). However, the profitability of these forests could be improved by exploiting other uses of the wood such as in industry, production of decorative objects, etc.

Improving the profitability of wood and resin production, along with the need to preserve and improve Tunisian thuya stands, which play a key ecological role in arid or semiarid environments, necessitates the development of models for quantitatively assessing the production of wood at different forest sites and under different management schedules to ensure a sustained yield. A first step towards this objective was the development of tree-level models relating common variables measured during forest inventories, such as breast height diameter, with relevant tree attributes like total tree height, crown diameter, height to crown base, stem taper curve and tree volume (Calama et al. 2012). The diameter-growth dynamics of the trees was studied by developing a distance-independent individual tree diameter increment model (Sghaier et al. 2013). A stand-level growth and yield model has recently been developed consisting of a site index sub-model and a system of compatible stand attribute equations present-

ed in the form of a stand density management diagram (Sghaier et al. 2015). The next step would be to characterize the stand diameter structure.

Fonseca et al. (2009) state that detailed information on the variability of tree diameters in natural and altered stands is key to sustainable forest management and is needed to assess carbon sequestration in forest ecosystems. Tree diameter distributions play an important role in stand management. For example, data regarding the number of trees per diameter class in a stand where a given silvicultural treatment has been applied are of particular interest because the diameter size will determine the industrial use of the wood and thus the price of the different products (Gorgoso et al. 2007). Diameter distribution data at stand level also provides a more scientific basis for forest managers to choose which trees should be harvested (Sghaier & Palm 2002).

Different types of parametric density functions have been used to describe tree diameter distribution in forest stands, including the Normal (Nanang 1998), Lognormal (Bliss & Reinker 1964, Nanang 1998), Gamma (Nelson 1964), Beta (Clutter & Bennett 1965, Palahi et al. 2007), Johnson's SB (Hafley & Schreuder 1977, Palahi et al. 2007, Fonseca et al. 2009) and the Weibull distribution (Bailey & Dell 1973, Palahi et al. 2007). Mateus & Tomé (2011) used Johnson's distribution for modeling the diameter distribution of first-rotation eucalyptus plantations in Portugal. Gorgoso et al. (2012) compared the accuracy of the Weibull, Johnson's SB and beta distributions for modeling even-aged stands of *Pinus pinaster* Ait., *Pinus radiata* D. Don and *Pinus sylvestris* L. in northwest Spain, and Binoti et al. (2012) evaluated the effectiveness of fatigue life, Frechet, Gamma, Generalized Gamma, Generalized Logistic, Loglogistic, Nakagami, Beta, Burr, Dagum, Weibull and Hyperbolic distributions for describing diameter distribution *Tectona grandis* L. stands subjected to thinning treatments at different ages in Brazil. Sghaier & Palm (2002) used Pearson's type I function for modeling diameter distribution of *Pinus halepensis* Mill. stands in Tunisia.

The objective of this study is to model the diameter distribution of the main *Tetraclinis articulata* stands in Tunisia. To achieve this objective, we compared the Normal distribution, given its ease of use, and the two-parameter Weibull distribution, since it has been shown to be the most suitable in many cases (Borders et al. 1987, Maltamo et al. 1995).

Material and methods

Study area and data collected

The natural *Tetraclinis articulata* stands at Jbel Lattrech forest in north-eastern Tunisia cover an area of approximately 4,000 hectares and are essentially mono specific and homogeneous. The climate is semi-arid, with an annual average precipitation of 393 mm, of which 95 % of rain falls between September and May. The average minimum temperatures for the coldest month (January) and maximum for the hottest month (August) are 8.4 and 30.5 °C, respectively (Ben Mansoura & Garchi 2001). The study zone is located on Oligocene sandstones where there are groupings of *Tetraclinis articulata – Lavandula stoechas*, dominated by *Cistus monspelliensis* L., *Genista aspalathoides* Lam., *Erica arborea* L., *Brachypodium ramosum* Roemer et Schultes, *Avena bromoides* Gouan and *Lupinis angustifolius* L. (Nabil 1989).

The data used to develop the diameter distribution model were collected in 2009 from 50 temporary circular sample plots of variable radius and established such that they cover a wide range of age, density and site quality combinations. The size of the plots ranged from 88 to 835 m², depending on the stand density, so as to include at least 40 measured trees per plot. The diameter at breast height of all trees in each plot (down to a minimum diameter of 5 cm) was measured to the nearest 0.1 cm. Additionally, the heights of a random sample of 11 trees per plot (including the 4 largest trees) were measured to estimate mean height. Dominant height was calculated from the four largest trees per plot, and age was estimated from stem analysis on fallen trees by counting the number of rings at 0.30 m (see Calama et al. 2012 for more details on data collection). Tab. 1 shows the main stand variables measured in the 50 plots.

Diameter distribution functions

We tested two distribution functions: the Normal function, known for its ease of use (Lejeune 1994), and the Weibull function, most frequently used because of its flexibility, simplicity and existence of a closed form solution for its CDF (Cao 2004, Merganic & Sterba 2006, Palahi et al. 2006, Gorgoso et al. 2007).

Normal distribution

The equation for the normal probability density function is (eqn. 1):

$$f(x)=\frac{1}{\sigma\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2\right]$$

where x is the random variable, and m and σ are its arithmetic mean and standard deviation, respectively.

The cumulative probability distribution of the normal function is (eqn. 2):

$$F(x)=\int_{-\infty}^{x} f(x)\,dx$$

The two parameters which define the normal distribution (arithmetic mean and the standard deviation) can be estimated by using the following expressions (Dagnelie 1998 – eqn. 3, eqn. 4):

$$\hat{m}=\frac{1}{n}\sum_{i=1}^{n} x_i$$

$$\hat{\sigma}=\sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i-\hat{m})^2}$$

where n indicates the number of trees per plot and x_i (cm) the diameter at breast height of each tree.

Weibull distribution

The variety of shapes that can be taken by the Weibull distribution makes it particularly suitable for representing tree diameter distribution. The equation for the Weibull probability density function is as follows (eqn. 5):

$$f(x)=\frac{c}{b}\left(\frac{x-a}{b}\right)^{c-1}\exp\left[-\left(\frac{x-a}{b}\right)^c\right]$$

for $x \ge a$, $a \ge 0$, $b > 0$, $c > 0$ or 0 if not. The cumulative probability distribution of the Weibull function is (eqn. 6):

$$F(x)=1-\exp\left[-\left(\frac{x-a}{b}\right)^c\right]$$

for $x > a$. This function is defined by three parameters: a is the location parameter, b is the scale parameter and c is the shape parameter.

Parameter a, commonly termed the location parameter, identifies the lower bound of the diameter distribution. For fixed values of b and c, changes in the parameter a simply shift the entire distribution along the x-axis. Parameter b is the scale parameter, and point $x = a + b$ corresponds approximately to the 63rd percentile of the distribution. Parameter c defines the shape of the Weibull distribution. When $c < 1$, the distribution has a reverse-J shape. In the

Tab. 1 - Summary of main stand variables for the sample used for modelling (Sghaier et al. 2015).

Variables	Mean	STD	Min	Max
Density (stems ha^{-1})	1860	865	479	4533
Age (years)	57	10	36	77
Site index (m)	5.06	1.08	2.83	7.78
Basal area (m^2 ha^{-1})	12.90	6.39	2.32	27.74
Quadratic mean diameter (cm)	9.41	1.55	6.32	12.95
Dominant diameter (cm)	12.76	2.32	7.85	17.43
Dominant height (m)	6.52	1.38	4.20	11.00
Mean height (m)	5.46	1.09	3.61	9.17
Volume (m^3 ha^{-1})	36.14	22.13	5.16	111.61

special case where $c = 1$, the Weibull distribution is reduced to the exponential distribution. When $c > 1$, the distribution is mound-shaped, approximately equivalent to the Normal distribution for $c = 3.6$. When c is between 1 and 3.6 the Weibull distribution is positively skewed, and is negatively skewed when c is greater than 3.6 (Bailey & Dell 1973).

The most frequently used methods to estimate the parameters of the Weibull distribution, are the maximum likelihood method (Jonhson & Kotz 1970) and the method of moments (Burk & Newberry 1984). Parameter a can be considered the smallest possible diameter in the stand and thus should be between zero and the minimum observed value in some cases (Bailey & Dell 1973). In some studies, the parameter a is arbitrarily fixed at 0.5 d_{min} (Lei 2008) or at zero (Gorgoso et al. 2007, Sevillano Marco et al. 2009), thus reducing the function to a two-parameter Weibull distribution which is easier to model and provides similar results to those of the three-parameter Weibull function, at least for even-aged, single-species stands (Maltamo et al. 1995, Mabvurira et al. 2002).

The two-parameter Weibull function (with the location parameter $a = 0$) was considered in this study as follows (eqn. 7):

$$f(x)=\frac{c}{b}\left(\frac{x}{b}\right)^{c-1}\exp\left[-\left(\frac{x}{b}\right)^{c}\right]$$

where x is the random variable, b the scale parameter and c the shape parameter that controls the skewness.

Maximum likelihood estimator (MLE)

The maximum likelihood method is a commonly used procedure for the Weibull distribution in forestry because it has certain desirable properties (Lei 2008). Estimation of the parameters using maximum likelihood has been found to produce consistently better goodness-of-fit statistics compared to other methods, but it also puts the greatest demands on the computational resources (Cao & McCarty 2006). If we consider the Weibull probability density function given in eqn. 7, then the likelihood function (L) will be (eqn. 8):

$$L(x_{1},...,x_{n};b,c)=\prod_{i=1}^{n}\frac{c}{b}\left(\frac{x_{i}}{b}\right)^{c-1}\exp\left[-\left(\frac{x_{i}}{b}\right)^{c}\right]$$

Taking the logarithms from eqn. 8, differentiating with respect to b and c respectively, and satisfying the following equations (Condés 1997, Nanos & Montero 2002, Eerikäinen & Maltamo 2003 – eqn. 9, eqn. 10):

$$c=\left[\frac{\sum_{i=1}^{n}x_{i}^{c}\log(x_{i})}{\sum_{i=1}^{n}x_{i}^{c}}-\frac{1}{n}\sum_{i=1}^{n}\log(x_{i})\right]^{-1}$$

$$b=\left(\frac{1}{n}\sum_{i=1}^{n}x_{i}^{c}\right)^{c-1}$$

where n equals the number of sample observations in a Weibull distribution and x_i the diameter of each tree. The value of c must be obtained from eqn. 9 by using standard iterative procedures and then it is used in eqn. 10 to obtain b.

Method of moments (MOM)

The method of moments is another technique commonly used for parameter estimation. In the Weibull distribution, the k moment readily follows from eqn. 7 and is (Lei 2008 – eqn. 11):

$$m_{k}=\left(\frac{1}{b}\right)^{k/c}\Gamma\left(1+\frac{k}{c}\right)$$

where Γ (Gamma function) is $\Gamma(s) = \int_0^{\infty}x^{s-1}e^{-x}dx$ ($s>0$). Then from eqn. 11, we can find the first and the second moment as follows (eqn. 12, eqn. 13):

$$m_{1}=\mu=\left(\frac{1}{b}\right)^{1/c}\Gamma\left(1+\frac{1}{c}\right)$$

$$m_{2}=\mu^{2}+\sigma^{2}=\left(\frac{1}{b}\right)^{2/c}\Gamma\left(1+\frac{2}{c}\right)$$

which gives (eqn. 14):

$$\sigma^{2}=m_{2}-\mu^{2}=\left(\frac{1}{b}\right)^{2/c}\left[\Gamma\left(1+\frac{2}{c}\right)-\Gamma^{2}\left(1+\frac{1}{c}\right)\right]$$

where σ^2 is the variance of tree diameters in a plot, and m_1, m_2 are the arithmetic and quadratic mean diameter in a plot, respectively.

When σ^2 is divided by the square of m_1, the expression for obtaining c is (eqn. 15):

$$\frac{\sigma^{2}}{\mu^{2}}=\frac{\Gamma\left(1+\frac{2}{c}\right)-\Gamma^{2}\left(1+\frac{1}{c}\right)}{\Gamma^{2}\left(1+\frac{1}{c}\right)}\Rightarrow$$

$$\sigma^{2}=\frac{\bar{d}^{2}}{\Gamma^{2}\left(1+\frac{1}{c}\right)}\cdot\left[\Gamma\left(1+\frac{2}{c}\right)-\Gamma^{2}\left(1+\frac{1}{c}\right)\right]$$

In order to estimate b and c, we need to calculate the arithmetic mean diameter $\bar{d}$ and the variance σ^2 of the observed distribution and obtain the estimator of c in eqn. 15. Eqn. 15 was resolved by an iterative procedure. When the value of the location parameter (a) is zero, the scale parameter (b) can then be calculated directly using the following equation (Gorgoso et al. 2007, Sevillano Marco et al. 2009 – eqn. 16):

$$b=\frac{\bar{d}}{\Gamma\left(1+\frac{1}{c}\right)}$$

where $\bar{d}$ is the arithmetic mean diameter.

Method comparison

To determine whether the adjusted theoretical distribution provides a good representation of what is observed, we used the chi-square test (χ^2). This test requires some extreme classes to be grouped when there are an insufficient number of observations (Dagnelie 1998). For this reason, the following goodness-of-fit statistics were computed for each method: the mean bias, which reflects the deviation of the model with respect to observed values and the root mean-square error (RMSE), which is a measure of the precision of the estimates.

For the calculation of these statistics, the observed value Y_i and the theoretical value $\hat{Y}_i$ were calculated for each diameter class per plot (2 cm diameter class). In addition, the error index (Reynolds et al. 1988) was calculated (eqn. 17):

$$EI_{i}=\sum_{i=1}^{k}\left|Y_{ik}-\hat{Y}_{ik}\right|$$

where Y_i and $\hat{Y}_i$ are the observed and predicted number of trees in diameter class k for the i-th plot. The sum includes all diameter classes in the i-th plot. In this error index, we can multiply the absolute difference between the observed and predicted number of trees per diameter class by the volume, using an equation developed by Sghaier et al. (2015): v = 2.182 · 10⁻² · d²·⁸⁹, R² = 0.917. The diameter used in this equation corresponds to the middle diameter class. In this case (eqn. 18):

$$EI_{i}=\sum_{i=1}^{k}\left(\left|Y_{ik}-\hat{Y}_{ik}\right|\cdot v_{k}\right)$$

This index can also be expressed relatively as a percentage by dividing it by the total volume of the observed distribution in the plot (Lejeune 1994 – eqn. 19).

$$EI_{i}^{'}=\frac{EI_{i}}{\sum_{i=1}^{k}Y_{i}v_{i}}\cdot100$$

The latter error index was systematically calculated for each plot in this study.

Parameter prediction: modeling distribution parameters with stand variables

In order to predict the parameters of the distribution functions directly from the stand variables using simple regression models, correlation analyses were carried out by estimating Pearson's correlation coefficients between the estimated parameters corresponding to each plot and different stand variables. The stand variables chosen were: age (A), density (N), dominant height (H_d), mean height (H_m), quadratic mean diameter (d_g), arithmetic mean diameter ($\bar{d}$), 25% percentile (P_{25}), median (P_{50}), 75% percentile (P_{75}) of the stand, and natural logarithm transformations for the quotient of the 25% percentile, the 50% percentile, the 75% percentile and the mean diameter by the quadratic mean diameter (LP_{25}, LP_{50}, LP_{75}, and Ld, respectively).

To predict the fitting parameters from the yield table using the Normal and Weibull distribution, the mean diameter and the variance must be known (first and second order moments of the distribution, respectively). The variance can be directly obtained from the arithmetic and the qua-

Tab. 2 - Summary statistics of the estimated parameters for the studied distributions (50 plots). (Sigma): $\hat{\sigma}$; (predicted mean): $\hat{\mu}$.

Distribution	Method	Parameter	Mean	Minimum	Maximum	CV%
Weibull	MLE	b	10.01	6.680	13.794	16.5
		c	4.134	3.137	7.087	18.6
	MOM	b	9.986	6.657	13.777	16.6
		c	4.418	3.077	7.410	20.8
Normal	-	Predicted mean	9.087	6.240	12.550	16.1
		Sigma	2.427	1.011	3.862	26.3

Tab. 3 - Correlations between stand variables and parameters estimated with the three studied distributions. (***): P<0.001; (**): P<0.01; (*): P<0.05; (A): age (years); N: density (trees ha^{-1}); H_d: dominant height (m); H_m: mean height (m); d_g: quadratic mean diameter (cm); ($\bar{d}$): mean diameter (cm); P_{25}: 25% percentile (cm); P_{50}: 50% percentile (cm); P_{75}: 75% percentile (cm); LP_{25}: ln(P_{25}/d_g); LP_{50}: ln(P_{50}/d_g); LP_{75}: ln(P_{75}/d_g); Ld=ln($\bar{d}/d_g$); ln: natural logarithm.

Variable	Normal distribution	Weibull distribution			
		Maximum Likelihood		Moments	
		b	c	b	c
A	0.088	0.218	0.176	0.221	0.110
N	-0.054	-0.170	-0.066	-0.174	-0.081
H_d	0.665***	0.667***	-0.368**	0.670***	-0.464***
H_m	0.720***	0.762***	-0.335*	0.763***	-0.456***
d_g	0.872***	0.999***	-0.380**	0.999***	-0.511***
$\bar{d}$	0.844***	0.998***	-0.336*	0.998***	-0.469**
P_{25}	0.648***	0.923***	-0.109	0.923***	-0.226
P_{50}	0.768***	0.970***	-0.231	0.971***	-0.367**
P_{75}	0.857***	0.976***	-0.359*	0.976***	-0.505***
LP_{25}	-0.617***	-0.253	0.742***	-0.254	0.783***
LP_{50}	-0.370**	-0.064	0.602***	-0.061	0.582***
LP_{75}	0.203	0.203	-0.042	0.206	-0.155
Ld	-0.811***	-0.463***	0.884***	-0.459***	0.908***

dratic mean diameter (Frazier 1981) by the following expression (eqn. 20):

$$\sigma^2 = d_g^2 - \bar{d}^2$$

The mean diameter can be modeled with eqn. 21, which ensures the prediction of $\bar{d}$ is lower than the quadratic mean diameter (d_g – eqn. 21):

$$\bar{d} = d_g - e^{X\beta}$$

where X is a vector of independent stand variables in a fixed instant and β is the vector of parameters to be estimated (Gorgoso et al. 2007, Sevillano Marco et al. 2009).

To select the model, the bias, the mean square error and the adjusted determination coefficient were computed.

Results

Tab. 2 shows summary descriptive statistics for the estimated parameters.

In order to adjust the models to predict the parameters of the theoretical distribu-

tions using the stand variables, Pearson's linear correlations for estimated parameters were calculated (Tab. 3).

The highest Pearson correlation between the parameters of Normal and Weibull distribution and the stand variables was obtained with d_g and its natural logarithm transformations. The values obtained for Pearson's linear correlation between the Ld parameter and the two estimated parameters of the Weibull distribution using the Maximum Likelihood and moments approaches were -0.463 and -0.459 for parameter b and 0.884 and 0.908 for parameter c. In the case of the shape parameter (c), the highest values obtained for Pearson's linear correlation were for the variable Ld (Tab. 3). The values obtained for the coefficients of determination were 91% and 96% for models that relate the parameter c to the variable Ld (Tab. S1 in Supplemetary material).

As regards the variance of the Normal distribution and the scale parameter (b) for the Weibull distribution, the results also show good linear relationships between these parameters and the measured stand variables. It appears that there is a logarithmic relation between the variance and quadratic mean diameter (d_g). The coefficient of determination was 78% (Tab. S1 in Supplementary material).

Note that the mean diameter is needed to estimate the shape parameter (c) in both parameter prediction methods of the Weibull distribution as well as in the Normal distribution. Since the yield tables generally only give the quadratic mean diameter (d_g), an equation was employed to predict the mean diameter from the quadratic mean diameter (d_g), the mean height (H_m) and the age (A) of the stand (Tab. S2 in Supplementary material).

According to the fitted prediction equation of the mean diameter (Tab. S2 in Supplementary material), the mean quadratic diameter (d_g), the mean height (H_m) and the age of the stand (A) must be known. Of these three parameters, only the mean height (H_m) cannot be directly obtained from yield tables and was therefore estimated by the fitting equation (eqn. 22) using the dominant height (H_d) disaggregated from yield tables as an independent variable (eqn. 22):

$$H_m = 0.8307 \cdot H_d$$

with R^2 = 0.989 and CV% = 11.03. The expected theoretical values for each plot are calculated using the three studied distributions with the two parameter prediction methods (estimated and prediction parameters). The observed, estimated and predicted values were grouped for each plot in 2 cm diameter classes. The results of the different comparison criteria are shown in Tab. 4.

In the parameter estimation, the Normal distribution is the most suitable for all the comparison criteria evaluated, with the

Tab. 4 - Bias, mean absolute error, residual mean square error and mean error index in number of trees for the three compared distributions (Weibull MLE, Weibull MOM and Normal) and the two methods of construction (parameter estimation and parameter prediction).

Methods	Comparison criteria	Weibull		Normal
		Maximum Likelihood	Moments	
Parameters estimation	Bias	0.0097	0.0068	0.0185
	MAE	1.7140	1.7145	1.6197
	RMSE	5.2173	5.3054	4.7749
	EI'%	32.61	33.40	31.42
Parameters prediction	Bias	0.0164	0.0063	0.0220
	MAE	1.7180	1.7592	1.8958
	RMSE	5.3606	5.6220	6.1839
	EI'%	32.25	34.03	35.13
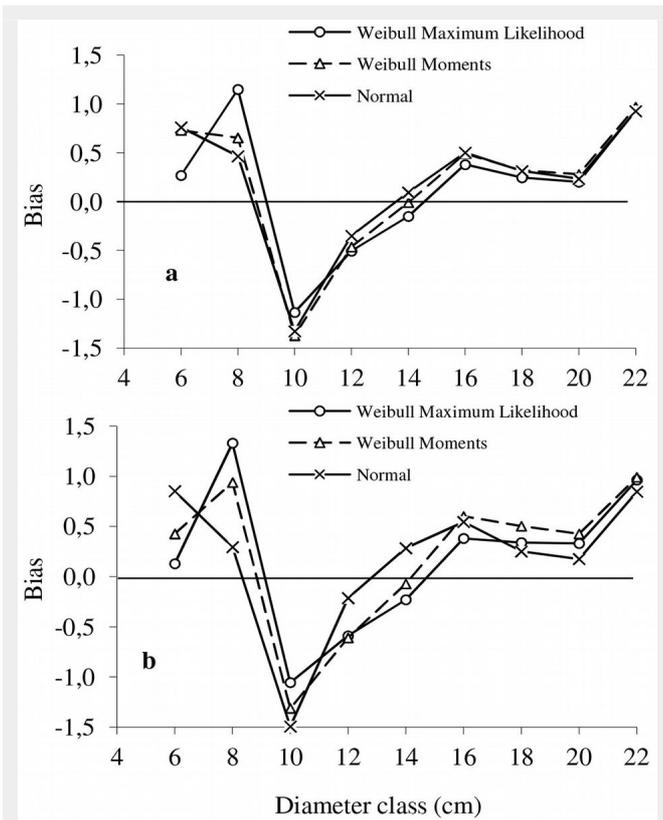
Fig. 1 - Values of bias in number of trees in each diameter class obtained using two fitting methods of the two-parameter Weibull distribution and Normal distribution. (a) Parameter estimation; (b): parameter prediction.
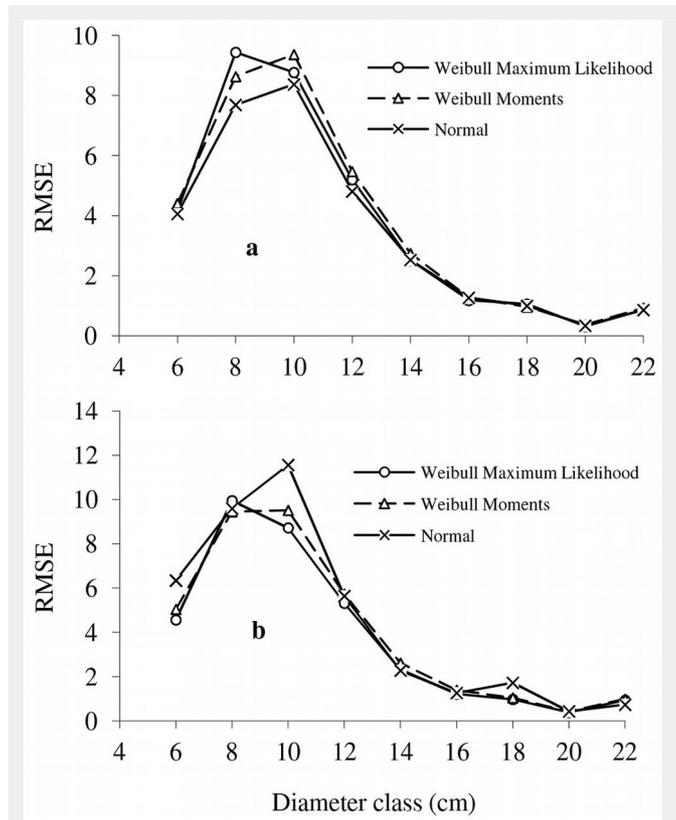
Fig. 2 - Values of mean square error (RMSE) in number of trees for each diameter class obtained using the two fitting methods of the two-parameter Weibull distribution and Normal distribution. (a) Parameter estimation; (b) parameter prediction.

exception of bias. Furthermore, the two parameter estimation approaches of the Weibull function gave similar results, although the maximum likelihood approach (MLE) was found to be slightly better.

In contrast to the parameter prediction in the first model construction approach (parameter estimation), the results obtained for parameter prediction revealed that the Weibull distribution was better for all the comparison criteria used. In this case, the maximum likelihood method also provided the best results, with the exception of the bias, for which the method of moments gave the lowest value.

In addition to comparing the distributions according to the Reynolds' index error (*EI* %), which is calculated for each plot, an analysis of variance (ANOVA) was performed for each model construction method. The results obtained reveal that there is no significant difference between the three distributions in each of the two model construction methods ($\alpha = 0.75$ for parameter estimation and $\alpha = 0.57$ for parameter prediction, respectively).

The Weibull distribution, using the method of moments, occupied the same intermediate position with the exception of the bias, for which it gave the lowest value in both cases (estimated and prediction parameters). To analyze these results in greater depth, some additional graphs were produced. The first two, (Fig. 1 and

Fig. 2), present the values of Bias and RMSE in each diameter class. Fig. 3 shows the mean value of the error index (*EI* %) in each density class.

According to the values for bias (Fig. 1), all the distributions underestimated the frequency of trees in the smallest and largest diameter classes, and overesti-
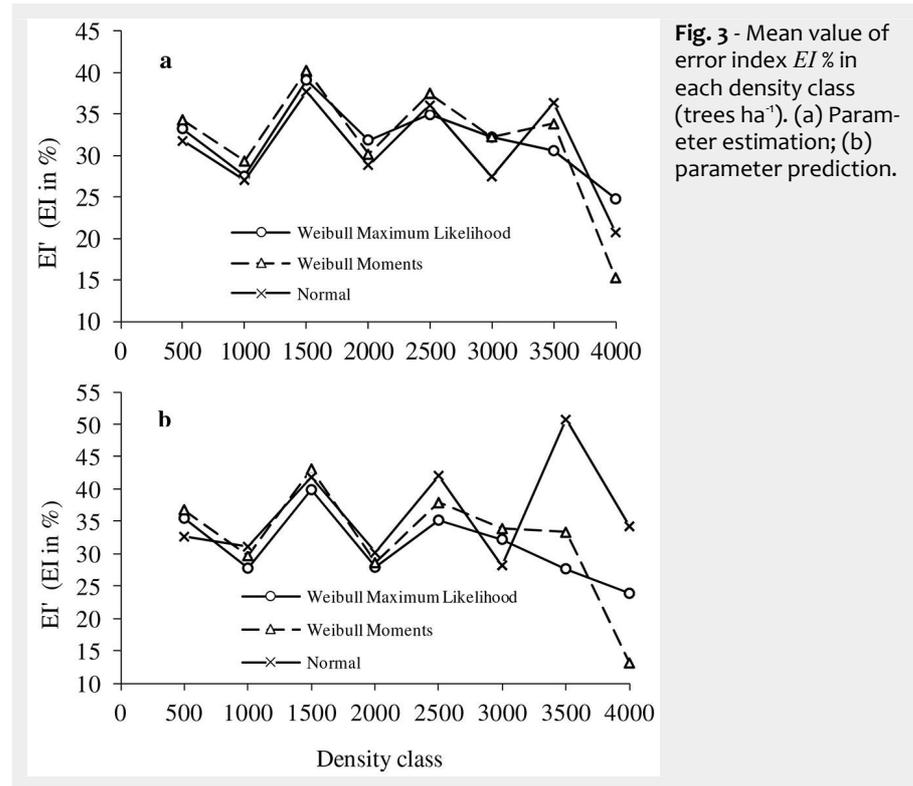
Fig. 3 - Mean value of error index *EI* % in each density class (trees ha⁻¹). (a) Parameter estimation; (b) parameter prediction.

Tab. 5 - Values of Weibull PDF parameters at 20, 40, 60, 80 and 100 years of age for stands with SDI < 400 and SI = 7 m (yield table).

Stand parameters					Weibull PDF parameters	
Age	N	H_m	D_g	D_m	b	c
20	1909	3.43	7.21	6.92	7.51	3.65
40	1400	5.81	10.1	9.70	10.84	3.65
60	1166	7.74	12.12	11.61	12.64	3.58
80	1024	9.33	13.67	13.07	13.83	3.53
100	925	10.26	14.91	14.31	14.69	3.64

mated them for the 10 cm diameter class under both parameter construction methods. With the exception of the 8 cm diameter class, for which the Weibull maximum likelihood method gives the highest value, the distributions compared present a similar level of precision, though the Normal distribution is slightly better, especially for parameter prediction.

As regards the mean square error (Fig. 2), although there are slight differences in the smallest diameter classes (lower or equal to 10 cm), the distributions gave similar results for diameter classes of 12 cm and above, being the Weibull distributions slightly superior in the case of parameter prediction.

For the Reynolds error index (*EI %*), Fig. 3 shows the variation of the mean value of this criterion according to stand density class (trees ha^{-1}). For both model construction approaches, the Weibull distribution, using the maximum likelihood method to

estimate the parameters, appears to give the most stable results of the three studied methods. In addition to its stability across all the density classes, this method also provides the most accurate results for the parameter prediction approach (Fig. 3).

According to the distribution of trees per diameter class in each of the 50 measured plots, three types of distribution shape were observed (symmetric distribution, right dissymmetric distribution and reverse-J distribution). In order to better appreciate the capacity of the adjusted models to predict the number of trees per diameter class at plot level, one representative plot per distribution type was selected. Graphs for estimated *vs.* observed values were constructed for a symmetric distribution (Fig. S1 in Supplementary material), a right skewed distribution (Fig. S2 in Supplementary material) and a reverse-J shaped distribution (Fig. S3 in Supplementary material) for the parameter estimation

and parameter prediction methods. The three figures reveal that all the studied models behave similarly, though the Weibull function appears to be marginally better than the others, in particular for the parameter prediction approach.

The use of the model for predicting the distribution of trees by diameter class

The diameter distribution models can be used independently by measuring a number of stand variables, or together with the yield table, which provides information that allows to predict the parameters of the distribution employed. The example which is provided here uses information from the yield table elaborated as part of this study for modeling the growth and production associated with the management and conservation of *Tetraclinis articulata* forests in Tunisia (Sghaier et al. 2015). Tab. 5 presents the predicted Weibull PDF (probability density function) parameters for different ages of the best site quality (H_d = 7 m at 50 years of age) in the case of low stand density (Reineke stand density index: SDI < 400).

Fig. 4 shows the curves associated with these distributions and Tab. 6 represents the distribution of trees (in number and as a percentage) per diameter class for various stand ages.

Based on the tree-level model attributes presented in Calama et al. (2012), which includes height-diameter relationship, a crown attributes model and a stem curve equation, it is possible to obtain information about the distribution of end-use timber volumes, or height distribution by diameter class.

Discussion

Our Pearson's correlations analysis results are very similar to the findings of Gorgoso et al. (2007) for *Betula alba* stands in northwest Spain, using the two-parameter Weibull function, except for the parameter *Ld*, which expresses the logarithm of the mean diameter divided by the quadratic mean diameter. In addition, the good linear correlations which are obtained between the scale parameter (*b*) of the Weibull function and the stand characteristics are analogous to those observed by Campos & Leite (2009), Leite et al. (2010) and Binoti et al. (2012). Linear models that relate *b* to the logarithm of the quadratic mean diameter (d_g) show a very high determination coefficient. Similar linear models for this relationship, with values of the adjusted determination coefficient close to 99%, were obtained by Alvarez González (1997) for *Pinus pinaster* in Galicia and García-Guëmes et al. (2002) for *Pinus pinea* stands in Valladolid (Spain).

The results obtained in this study reveal that the Normal distribution, though less flexible, provides results which are as satisfactory as those of the Weibull distribution. Lejeune (1994) reported similar findings for *Picea abies* in Belgium. However, the Wei-

Fig. 4 - Curves of diametric distribution at 20, 40, 60, 80 and 100 years by diameter (DBH, cm) for stands with SDI < 400 and SI =7 m (density of probability).
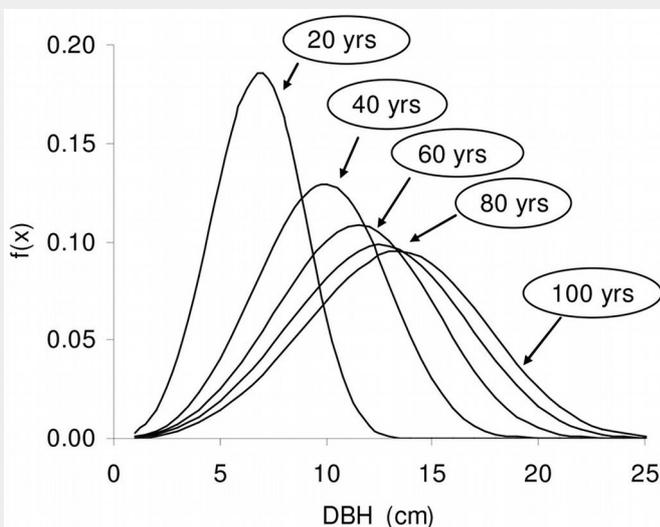
Tab. 6 - Distribution of trees at 20, 40, 60, 80 and 100 years by DBH class for stands with SDI < 400 and SI =7 m.

DBH cm	Number of trees (Age)					Percentage (%) (Age)				
	20	40	60	80	100	20	40	60	80	100
< 5	521	112	57	38	25	27	8	5	4	3
6-10	1327	714	412	284	210	70	51	36	28	23
11-15	61	540	552	474	416	3	39	47	46	45
16-20	0	34	141	211	242	0	2	12	20	26
21-25	0	0	4	18	31	0	0	0	2	3
26-30	0	0	0	0	1	0	0	0	0	0
Total	1909	1400	1166	1025	925	100	100	100	100	100

bull distribution using the maximum likelihood method to estimate parameters appears to provide a slightly superior method in terms of *RMSE, MAE* and *EI%*, especially with the parameter prediction approach.

A number of authors have reported that the two-parameter Weibull function provides the most simple and accurate approach for modelling diameter distributions, including Gorgoso (2003), and Gorgoso et al. 2007) for birch stands (*Betula alba* L.) in north-western Spain, Maltamo et al. (1995) for *Pinus sylvestris* and *Picea abies* (L.) Karst. stands in Finland, Alvarez González (1997) for *Pinus pinaster* stands, and Condés (1997) for different species in Spain. Furthermore, the two-parameter Weibull function has been used in a large number of related studies due to its flexibility and high correlation of its parameters with stand characteristics (Nogueira et al. 2005, Binoti et al. 2012). Therefore, given its flexibility and the very strong correlation between the PDF parameters and stand characteristics, the two-parameter Weibull distribution, using the maximum likelihood method, appears to be the most suitable for describing diameter distributions in this study.

Gorgoso et al. (2012) and Zhang et al. (2003) in their studies in north west Spain and the north-eastern part of North America, respectively, obtained better results with the 3-parameter Weibull distribution, using the maximum likelihood method rather than the moments approach.

Conclusion

In this study, a model has been developed for predicting the distribution of trees by diameter class using stand variables from *Tetraclinis articulata* forests in north-eastern Tunisia. The developed model, used in conjunction with the previously developed models for tree level attributes (Calama et al. 2012), the distance-independent individual tree diameter-increment model (Sghaier et al. 2013) and the stand based growth and yield model presented in Sghaier et al. (2015), constitutes a useful tool for the sustainable management of *Tetraclinis articulata* forests in north-east Tunisia.

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Supplementary Material

Tab. S1 - Regression equations retained for the parameter prediction using the stand variables (50 plots).

Tab. S2 - Parameter estimations and statistics used for modeling the mean diameter ($\bar{d}$) of the stand.

Fig. S1 - Observed *vs.* estimated values for the parameter estimation and parameter prediction approaches according to each test function for the plot number 36 with a symmetrical distribution of tree diameters.

Fig. S2 - Observed versus estimated values for the parameter estimation and parameter prediction method according to each test function for the plot number 50 with a right dissymmetrical distribution of tree diameters (positively skewed).

Fig. S3 - Observed versus estimated values for the parameter estimation and parameter prediction approaches according to each test function for the plot number 44 with a reverse-J distribution of tree diameters.

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