

Supplementary Material

Appendix 1 - The structure and values of the elements in vectors and matrices that form the system of motion equations.

The vector of generalized coordinates $\{\mathbf{q}\}$ of size 34 is formed from sub-vectors $[\mathbf{q}]_h$ of size 6, $[\mathbf{q}]_t$ of size 18, $[\mathbf{q}]_m$ of size 3, $[\mathbf{q}]_{hh}$ of size 4, and $[\mathbf{q}]_{as}$ of size 3 (eqn. 1):

$$\{\mathbf{q}\}^T = \{[\mathbf{q}]_h \quad [\mathbf{q}]_t \quad [\mathbf{q}]_m \quad [\mathbf{q}]_{hh} \quad [\mathbf{q}]_{as}\} \quad (1)$$

where T is the vector transposition operation.

The sub-vectors describe the individual components of the harvester and include generalized coordinates that correspond to these components. The sub-vector $[\mathbf{q}]_h$ describes the harvester as a whole. The sub-vector $[\mathbf{q}]_t$ describes the wheels of the harvester. The sub-vector $[\mathbf{q}]_m$ describes the manipulator. The sub-vector $[\mathbf{q}]_{hh}$ describes the harvester head with the log fixed in it. Finally, the sub-vector $[\mathbf{q}]_{as}$ describes the anchor system of the harvester. These sub-vectors have the following structure (eqn. 2, eqn. 3, eqn. 4):

$$\{\mathbf{q}\}_h^T = \{q_1 \quad q_2 \quad q_3 \quad q_4 \quad q_5 \quad q_6\} \quad (2)$$

$$\{\mathbf{q}\}_t^T = \{q_7 \quad q_8 \quad q_9 \quad q_{10} \quad q_{11} \quad q_{12} \quad q_{13} \quad q_{14} \quad q_{15} \quad q_{16} \quad q_{17} \quad q_{18} \quad q_{19} \quad q_{20} \quad q_{21} \quad q_{22} \quad q_{23} \quad q_{24}\} \quad (3)$$

$$\{\mathbf{q}\}_m^T = \{q_{25} \quad q_{26} \quad q_{27}\}; \quad \{\mathbf{q}\}_{hh}^T = \{q_{28} \quad q_{29} \quad q_{30} \quad q_{31}\}; \quad \{\mathbf{q}\}_{as}^T = \{q_{32} \quad q_{33} \quad q_{34}\} \quad (4)$$

The vectors of generalized velocities $\{\dot{\mathbf{q}}\}$ and generalized accelerations $\{\ddot{\mathbf{q}}\}$ are structurally equivalent to the vector $\{\mathbf{q}\}$. The generalized coordinates q_k must be replaced by the corresponding generalized velocities $\dot{q}_k$ or generalized accelerations $\ddot{q}_k$.

The vector of generalized forces $\{Q\}$ of size 34 is formed from sub-vectors $[Q]_h$ of size 6, $[Q]_t$ of size 18, $[Q]_m$ of size 3, $[Q]_{hh}$ of size 4, and $[Q]_{as}$ of size 3 (eqn. 5):

$$\{Q\}^T = \{[Q]_h \quad [Q]_t \quad [Q]_m \quad [Q]_{hh} \quad [Q]_{as}\} \quad (5)$$

These sub-vectors have the following structure (eqn. 6, eqn. 7, eqn. 8):

$$\{Q\}_h^T = \{Q_1 \quad Q_2 \quad Q_3 \quad Q_4 \quad Q_5 \quad Q_6\} \quad (6)$$

$$\{Q\}_t^T = \{Q_7 \quad Q_8 \quad Q_9 \quad Q_{10} \quad Q_{11} \quad Q_{12} \quad Q_{13} \quad Q_{14} \quad Q_{15} \quad Q_{16} \quad Q_{17} \quad Q_{18} \quad Q_{19} \quad Q_{20} \quad Q_{21} \quad Q_{22} \quad Q_{23} \quad Q_{24}\} \quad (7)$$

$$\{Q\}_m^T = \{Q_{25} \quad Q_{26} \quad Q_{27}\}; \quad \{Q\}_{hh}^T = \{Q_{28} \quad Q_{29} \quad Q_{30} \quad Q_{31}\}; \quad \{Q\}_{as}^T = \{Q_{32} \quad Q_{33} \quad Q_{34}\} \quad (8)$$

Vector of projections of operational forces F_x, F_y, F_z on the coordinate axes (eqn. 9):

$$\{F\}^T = \{F_x \quad F_y \quad F_z\} \quad (9)$$

The diagonal matrix $[A]$ of coefficients for the generalized coordinates of size 34×34 is formed by combining diagonal sub-matrices $[A]_h, [A]_t, [A]_m, [A]_{hh}$, and $[A]_{as}$ (eqn. 10):

$$[A] = \begin{bmatrix} [A]_h & [0]_{6-18} & [0]_{6-3} & [0]_{6-4} & [0]_{6-3} \\ [0]_{18-6} & [A]_t & [0]_{18-3} & [0]_{18-4} & [0]_{18-3} \\ [0]_{3-6} & [0]_{3-18} & [A]_m & [0]_{3-4} & [0]_{3-3} \\ [0]_{4-6} & [0]_{4-18} & [0]_{3-6} & [A]_{hh} & [0]_{3-3} \\ [0]_{3-6} & [0]_{3-18} & [0]_{3-3} & [0]_{3-4} & [A]_{as} \end{bmatrix}, \quad (10)$$

where $[0]_{m-n}$ is the zero matrix of size $m \times n$.

Elements of the main diagonal of the diagonal sub-matrix $[A]_h$ of size 6×6 (eqn. 11):

$$[A]_h = \text{diag} \left[-\frac{3c_{rwt}c_{tx}}{3c_{tx}+2c_{rwt}}; 3c_{st}-\frac{c_r c_w}{c_r+c_w}; -3c_{st}-\frac{c_r c_w}{c_r+c_w}; -1.5c_{st}z_h; -c_{rwt}\frac{(B_h-x_h)^2}{L_r}; c_{tx} \sum_{k=1}^{k=1} (x_{tk}-x_h) \right], \quad (11)$$

where x_h, z_h are the coordinates of the mass center for the harvester itself, B_h is the distance to the mass center of the anchor winch, L_r is the distance to the anchor tree, c_{rwt} is the stiffness coefficient

of the "anchor tree-anchor rope-anchor winch" system, c_{st} is the stiffness coefficient of the "suspension-wheelset-soil" system in the vertical direction (along the axis y).

Elements of the main diagonal of the diagonal sub-matrix $[A]_t$ of size 18×18 (eqn. 12):

$$[A]_t = \text{diag}[-0.5c_{tx}; -0.5c_s; -0.5c_{tz}; -0.5c_{tx}; -0.5c_s; -0.5c_{tz}; -0.5c_{tx}; -0.5c_s; -0.5c_{tz}; \rightarrow \\ \rightarrow -0.5c_{tx}; -0.5c_s; -0.5c_{tz}; -0.5c_{tx}; -0.5c_s; -0.5c_{tz}; -0.5c_{tx}; -0.5c_s; -0.5c_{tz}]. \quad (12)$$

Elements of the main diagonal of the diagonal sub-matrix $[A]_m$ of size 3×3 (eqn. 13):

$$[A]_m = \text{diag}\left[-(6c_{tx} + c_{rwt}); c_{ty} \frac{2x_{t1} - x_{t2} - x_{t3}}{x_{t1} + x_m}; c_{tz} \frac{2x_{t1} - x_{t2} - x_{t3}}{x_{t1} + x_m}\right], \quad (13)$$

where x_{tk} , x_m are coordinates of the manipulator base and the mass center of the wheel mounted on the k -th axis.

Elements of the main diagonal of the diagonal sub-matrix $[A]_{hh}$ of size 4×4 (eqn. 14):

$$[A]_{hh} = \text{diag}\left[-(6c_{tx} + c_{rwt} + c_{mx}q_{m1}); 2c_{tgy} \frac{2x_{t1} - x_{t2} - x_{t3}}{x_{t1} + x_{hh}} - c_{my}; c_{tz} \frac{2x_{t1} - x_{t2} - x_{t3}}{x_{t1} + x_{hh}} - c_{mz}; -l_G\right], \quad (14)$$

where c_{mx} , c_{my} , c_{mz} are stiffness coefficients of the manipulator along the axes x , y , z , x_{hh} is the coordinate x of the mass center of the harvester head, l_G is the log suspension length.

Elements of the main diagonal of the diagonal sub-matrix $[A]_{as}$ of size 3×3 (eqn. 15):

$$[A]_{as} = \text{diag}\left[c_{rwt} + \frac{2}{3}c_{tx}; \frac{c_r c_{at}}{c_r + c_{at}}; 0.5 \frac{3c_{rwt} c_{tx}}{2c_{rwt} + 3c_{tx}} \left(\frac{H_L}{H_{at}}\right)^2 \left(3 - \frac{H_L}{H_{at}}\right)\right], \quad (15)$$

where H_L is the height of the anchor rope attachment to the anchor tree.

A square matrix $[B]$ of coefficients for generalized velocities of size 34×34 is formed by combining diagonal sub-matrices $[B]_h$, $[B]_t$, $[B]_{hh}$ and rectangular sub-matrices $[B]_{mt}$, $[B]_{ht}$, $[B]_{hht}$, $[B]_{ast}$, $[B]_{has}$ (eqn. 16):

$$[\mathbf{B}] = \begin{bmatrix} [\mathbf{B}]_h & [\mathbf{B}]_{ht} & [\mathbf{0}]_{6-3} & [\mathbf{0}]_{6-4} & [\mathbf{B}]_{has} \\ [\mathbf{0}]_{18-6} & [\mathbf{B}]_t & [\mathbf{0}]_{18-3} & [\mathbf{0}]_{18-4} & [\mathbf{0}]_{18-3} \\ [\mathbf{0}]_{3-6} & [\mathbf{B}]_{mt} & [\mathbf{0}]_{3-3} & [\mathbf{0}]_{3-4} & [\mathbf{0}]_{3-3} \\ [\mathbf{0}]_{4-6} & [\mathbf{B}]_{hht} & [\mathbf{0}]_{3-6} & [\mathbf{B}]_{hh} & [\mathbf{0}]_{3-3} \\ [\mathbf{0}]_{3-6} & [\mathbf{B}]_{ast} & [\mathbf{0}]_{3-3} & [\mathbf{0}]_{3-4} & [\mathbf{B}]_{as} \end{bmatrix}. \quad (16)$$

Diagonal sub-matrix $[\mathbf{B}]_h$ of size 6×6 (eqn. 17):

$$[\mathbf{B}]_h = \text{diag}[0; -6\beta_s; 0; 0; 0; 0]. \quad (17)$$

Diagonal sub-matrix $[\mathbf{B}]_t$ of size 18×18 (eqn. 18):

$$[\mathbf{B}]_t = \text{diag}[-2\beta_{tx}; -0.5\beta_s; -2\beta_{tz}; -2\beta_{tx}; -0.5\beta_s; -2\beta_{tz}; -2\beta_{tx}; -0.5\beta_s; -2\beta_{tz}; \rightarrow \\ \rightarrow -2\beta_{tx}; -0.5\beta_s; -2\beta_{tz}; -2\beta_{tx}; -0.5\beta_s; -2\beta_{tz}; -2\beta_{tx}; -0.5\beta_s; -2\beta_{tz}]. \quad (18)$$

Diagonal sub-matrix $[\mathbf{B}]_{hh}$ of size 4×4 (eqn. 19):

$$[\mathbf{B}]_{hh} = \text{diag}[0; -(\beta_{h1} + \beta_{h2} + \beta_{h3}); 0; -\beta_d l_G], \quad (19)$$

where β_d is the coefficient of viscous resistance of the harvester head damper.

Rectangular sub-matrix $[\mathbf{B}]_{ht}$ of size 6×18 (eqn. 20):

$$[\mathbf{B}]_{ht} = - \begin{bmatrix} \beta_{tx} & 0 & 0 & \beta_{tx} & 0 & 0 & \beta_{tx} & 0 & 0 \\ 0 & \beta_{ty} & 0 & 0 & \beta_{ty} & 0 & 0 & \beta_{ty} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \beta_{ty}(z_t + z_h) & 0 & 0 & \beta_{ty}(z_t + z_h) & 0 & 0 & \beta_{ty}(z_t + z_h) & 0 \\ 0 & 0 & \beta_{tz}(x_{t1} - x_h) & 0 & 0 & \beta_{tz}(x_{t1} - x_h) & 0 & 0 & \beta_{tz}(x_{t1} - x_h) \\ 0 & \beta_{tgy}(x_{t1} - x_h) & 0 & 0 & \beta_{tgy}(x_{t1} - x_h) & 0 & 0 & \beta_{tgy}(x_{t1} - x_h) & 0 \end{bmatrix} \rightarrow \\ \rightarrow \begin{bmatrix} \beta_{tx} & 0 & 0 & \beta_{tx} & 0 & 0 & \beta_{tx} & 0 & 0 \\ 0 & \beta_{ty} & 0 & 0 & \beta_{ty} & 0 & 0 & \beta_{ty} & 0 \\ 0 & 0 & \beta_{tz} & 0 & 0 & \beta_{tz} & 0 & 0 & \beta_{tz} \\ 0 & \beta_{ty}(z_t - z_h) & 0 & 0 & \beta_{ty}(z_t - z_h) & 0 & 0 & \beta_{ty}(z_t - z_h) & 0 \\ 0 & 0 & \beta_{tz}(x_{t1} - x_h) & 0 & 0 & \beta_{tz}(x_{t2} - x_h) & 0 & 0 & \beta_{tz}(x_{t3} - x_h) \\ 0 & \beta_{tgy}(x_{t1} - x_h) & 0 & 0 & \beta_{tgy}(x_{t2} - x_h) & 0 & 0 & \beta_{tgy}(x_{t3} - x_h) & 0 \end{bmatrix}. \quad (20)$$

Rectangular sub-matrix $[\mathbf{B}]_{mt}$ of size 3×18 (eqn. 21):

$$[B]_{mt} = - \begin{bmatrix} \beta_{tx} & 0 & 0 & \beta_{tx} & 0 & 0 & \beta_{tx} & 0 & 0 & \beta_{tx} & 0 & 0 & \beta_{tx} & 0 & 0 & \beta_{tx} & 0 & 0 \\ 0 & \beta_{tgy} & 0 & 0 & \beta_{tgy} & 0 & 0 & \beta_{tgy} & 0 & 0 & \beta_{tgy} & 0 & 0 & \beta_{tgy} & 0 & 0 & \beta_{tgy} & 0 \\ 0 & 0 & 0 & 0 & 0 & \beta_{tz} & 0 & 0 & \beta_{tz} & 0 & 0 & 0 & 0 & 0 & \beta_{tz} & 0 & 0 & \beta_{tz} \end{bmatrix} \quad (21)$$

Rectangular sub-matrix $[B]_{hht}$ of size 4×18 (eqn. 22):

$$[B]_{hht} = - \begin{bmatrix} \beta_{tx} & 0 & 0 & \beta_{tx} & 0 & 0 & \beta_{tx} & 0 & 0 & \beta_{tx} & 0 & 0 & \beta_{tx} & 0 & 0 & \beta_{tx} & 0 & 0 \\ 0 & 0 & 0 & 0 & \beta_{tz} & 0 & 0 & \beta_{tz} & 0 & 0 & \beta_{tz} & 0 & 0 & \beta_{tz} & 0 & 0 & \beta_{tz} & 0 \\ 0 & 0 & 0 & 0 & 0 & \beta_{tz} & 0 & 0 & \beta_{tz} & 0 & 0 & 0 & 0 & 0 & \beta_{tz} & 0 & 0 & \beta_{tz} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (22)$$

Rectangular sub-matrix $[B]_{ast}$ of size 3×18 (eqn. 23):

$$[B]_{ast} = - \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \beta_{tx} & 0 & 0 & \beta_{tx} & 0 & 0 & \beta_{tx} & 0 & 0 & \beta_{tx} & 0 & 0 & \beta_{tx} & 0 & 0 & \beta_{tx} & 0 & 0 \\ \beta_{tx} & 0 & 0 & \beta_{tx} & 0 & 0 & \beta_{tx} & 0 & 0 & \beta_{tx} & 0 & 0 & \beta_{tx} & 0 & 0 & \beta_{tx} & 0 & 0 \end{bmatrix} \quad (23)$$

Rectangular sub-matrices $[B]_{has}$ of size 6×3 and $[B]_{as}$ of size 3×3 (eqn. 24):

$$[B]_{has} = - \begin{bmatrix} 0 & 0 & \beta_{at} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}; \quad [B]_{as} = - \begin{bmatrix} 6\beta_{tx} & 0 & \beta_{at} \\ 0 & 0 & \beta_{at} \\ 0 & 0 & \beta_{at} \end{bmatrix} \quad (24)$$

A rectangular matrix $[C]$ consisting of coefficients for the projections of operational forces of size 34×3 is formed from the rectangular sub-matrices $[C]_h$, $[C]_m$, $[C]_{hh}$, and $[C]_{as}$ (eqn. 25):

$$[C] = \begin{bmatrix} [C]_h \\ [0]_{18 \times 3} \\ [C]_m \\ [C]_{hh} \\ [C]_{as} \end{bmatrix} \quad (25)$$

Rectangular sub-matrix $[C]_h$ of size 6×3 (eqn. 26):

$$[C]_h = \begin{bmatrix} -\cos q_{m1} & 0 & \sin q_{m1} \\ 0 & -1 & 0 \\ \sin q_{m1} & 0 & -\cos q_{m1} \\ 0 & z_{hh}-z_h & y_{hh}-y_h \\ (x_{hh}+x_h)\cos q_{m1}-(z_{hh}-z_h)\sin q_{m1} & 0 & (x_{hh}+x_h)\sin q_{m1}-(z_{hh}-z_h)\cos q_{m1} \\ y_{hh}-y_h & x_{hh}+x_h & 0 \end{bmatrix}. \quad (26)$$

Rectangular sub-matrix $[C]_m$ of size 3×3 (eqn. 27):

$$[C]_m = \begin{bmatrix} -\cos q_{m1} & 0 & \sin q_{m1} \\ 0 & -\frac{x_{t1}+x_{hh}}{x_{t1}+x_m} & 0 \\ \frac{x_{t1}+x_{hh}}{x_{t1}+x_m} \sin q_{m1} & 0 & \frac{x_{t1}+x_{hh}}{x_{t1}+x_m} \cos q_{m1} \end{bmatrix}. \quad (27)$$

Rectangular sub-matrices $[C]_{hh}$ of size 4×3 and $[C]_{as}$ of size 3×3 (eqn. 28):

$$[C]_{hh} = \begin{bmatrix} -\cos q_{m1} & 0 & \sin q_{m1} \\ 0 & -1 & 0 \\ \sin q_{m1} & 0 & \cos q_{m1} \\ l_G \cos \alpha_s & -l_G & 0 \end{bmatrix};$$

$$[C]_{as} = \begin{bmatrix} -1 & -(\sin \alpha_s + \mu_{gr} \cos \alpha_s) & 0 \\ -\cos q_{m1} & 0 & \sin q_{m1} \\ -\cos q_{m1} & 0 & \sin q_{m1} \end{bmatrix}. \quad (28)$$

A diagonal matrix $[E]$ of coefficients for generalized accelerations of size 34×34 is formed as (eqn. 29):

$$[E] = \begin{bmatrix} [E]_h & [0]_{6-18} & [0]_{6-3} & [0]_{6-4} & [0]_{6-3} \\ [0]_{18-6} & [E]_t & [0]_{18-3} & [0]_{18-4} & [0]_{18-3} \\ [0]_{3-6} & [0]_{3-18} & [E]_m & [0]_{3-4} & [0]_{3-3} \\ [0]_{4-6} & [0]_{4-18} & [0]_{3-6} & [E]_{hh} & [0]_{3-3} \\ [0]_{3-6} & [0]_{3-18} & [0]_{3-3} & [0]_{3-4} & [E]_{as} \end{bmatrix}. \quad (29)$$

Elements of the main diagonal of the diagonal sub-matrices $[\mathbf{E}]_h$ of size 6×6 , $[\mathbf{E}]_t$ of size 18×18 , $[\mathbf{E}]_m$ of size 3×3 , $[\mathbf{E}]_{hh}$ of size 4×4 и $[\mathbf{E}]_{as}$ of size 3×3 (eqn. 30, eqn. 31, eqn. 32, eqn. 33, eqn. 34):

$$[E]_h = \text{diag} [M_{h, \text{sum}} + M_w; \quad M_{h, \text{sum}} + M_w; \quad M_{h, \text{sum}} + M_w; \quad J_{hm_x}; \quad J_{hm_y}; \quad J_{hm_z}]; \quad (30)$$

[illegible]

$$[E]_m = \text{diag} \begin{bmatrix} M_m + M_G; & M_m + M_G; & M_m + M_G \end{bmatrix}; \quad (32)$$

$$[E]_{hh} = \text{diag} [M_G; M_G; M_G; J_{Gz}]; \quad (33)$$

$$[E]_{as} = diag [M_w; M_r; M_{at}], \quad (34)$$

where $M_{h,sum}$ is the total mass of the simulated system, including the harvester, manipulator, anchor system and log (excluding the anchor tree mass), $J_{hm x}$, $J_{hm y}$, $J_{hm z}$ are the total inertia moments of the harvester and manipulator when turning relative to the axes x , y , z .

The vector of the right-hand sides of the motion equations of size 34 has the following form (eqn. 35):

$$[G]^T = \{g_1, g_2, \dots, g_i, \dots, g_{34}\} = \{Q_1/e_{1,1}, Q_2/e_{2,2}, \dots, Q_i/e_{i,i}, \dots, Q_{34}/e_{34,34}\}, \quad (35)$$

where $e_{i,i}$ is the i -th diagonal element of the matrix $[\mathbf{E}]$.

Fig. S1 – Geometric model.

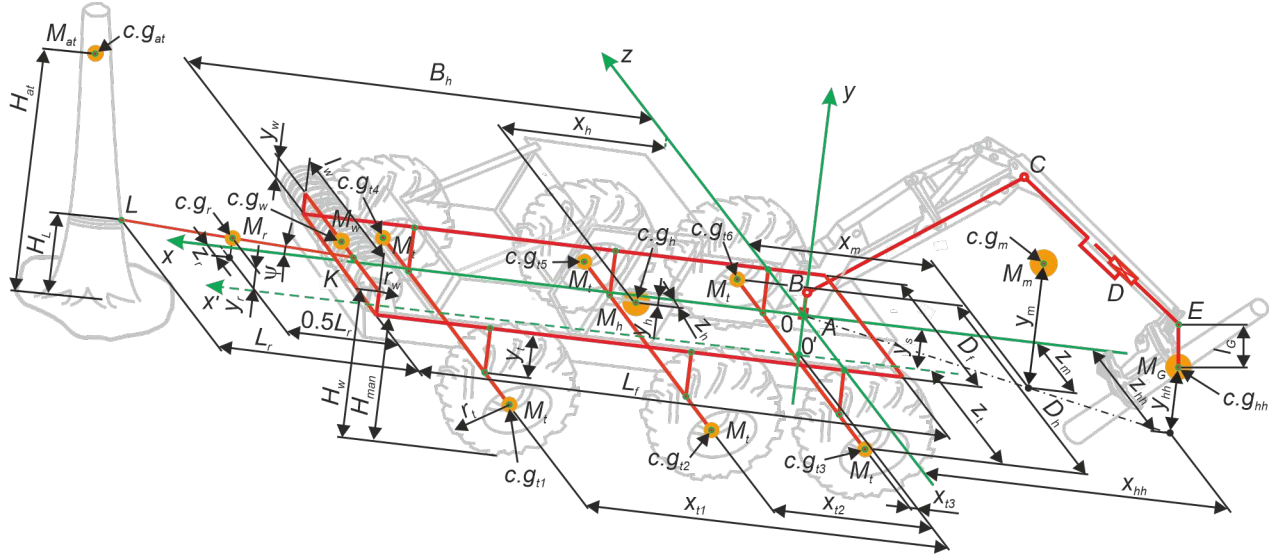


Fig. S2 – Generalized coordinates of the dynamic mathematical model.

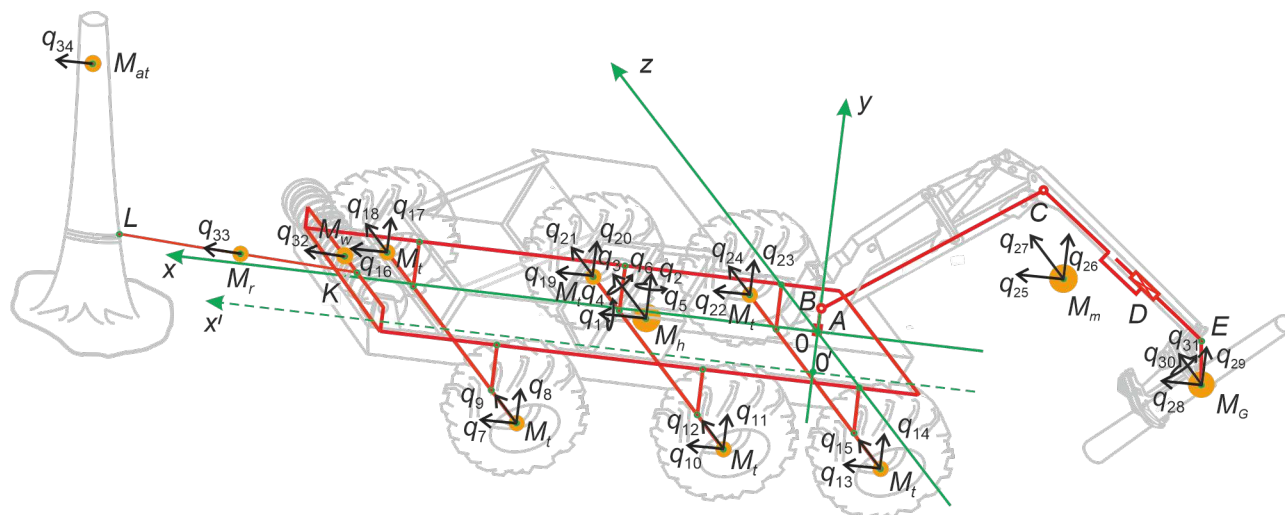


Fig. S3 – The dynamics of the mass center movement of an anchor tree.

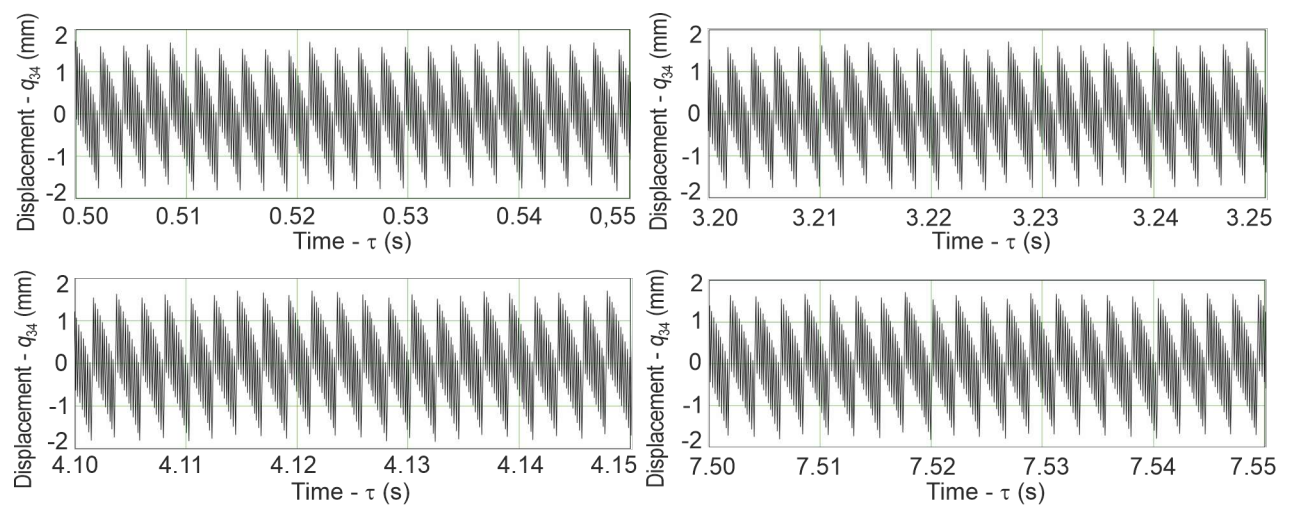


Fig. S4 – The dynamics of the anchor winch drum’s movement.

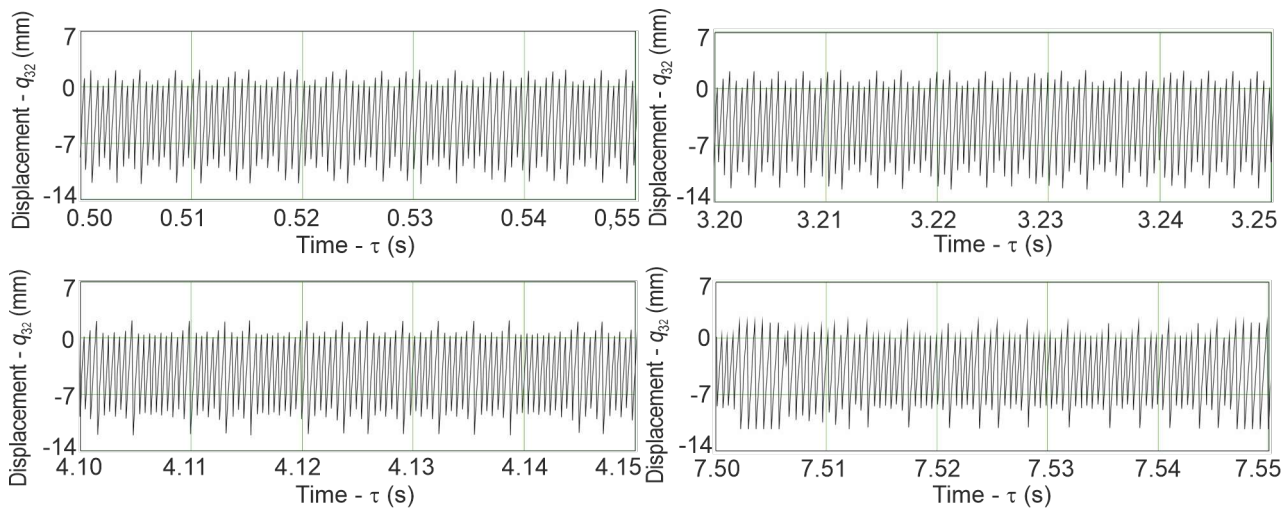


Fig. S5 – Dynamics of the mass center movement of the harvester head and the log along the x -axis.

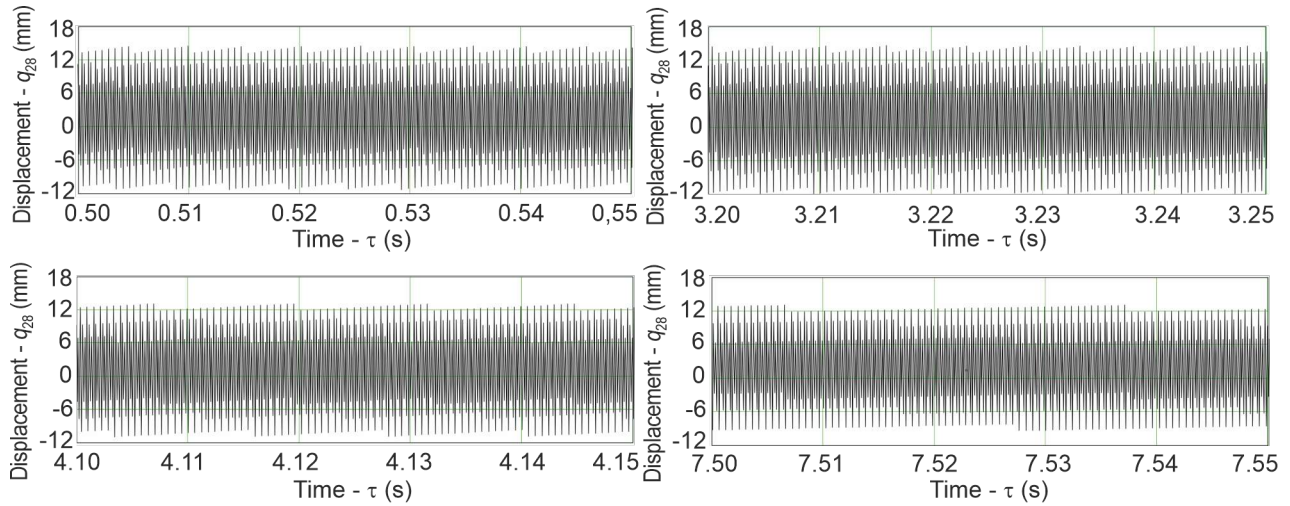


Fig. S6 – Histograms of the distribution of the ranges of relative share for the anchor’s rope tensile force (a), the mass center movement of the harvester head and the log along the x -axis (b), the anchor winch drum’s movement (c), and the mass center movement of an anchor tree (d).

