Supplementary Material

Appendix 1 - Theoretical derivation of calibration characteristics for length standards.

In the calculation of a rectangular area ($A = a \times b$), both side lengths a and b are typically measured with the same instrument, introducing a dependency between them. Consequently, the covariance between these measurements, denoted $u(a, b)$, must be included as an additional component of overall uncertainty (OIML G 19 2017). The strength of this dependence is quantified by the correlation coefficient, $r(a, b)$. Assuming equal standard uncertainties for both side lengths ($u(a) = u(b) = u(l)$), where $u(l)$ is the standard uncertainty of the length measurement) and assuming full positive correlation ($r = 1$), the combined standard uncertainty of the area can be determined using the following (eqn. A1):

$$u(A) = \sqrt{\begin{matrix}\left(W_a \cdot u(l)\right)^2 + \left(W_b \cdot u(l)\right)^2 + \\ 2 \cdot W_a \cdot W_b \cdot u(l)^2 \cdot r\end{matrix}} = (W_a + W_b)u(l) \quad \text{(A1)}$$

where $W_a = b$ and $W_b = a$ are the absolute sensitivity coefficients. From (eqn. A1), one can derive (eqn. A2):

$$u(l) = \frac{u(A)}{(W_a + W_b)} \quad \text{(A2)}$$

We assume that the standard uncertainty of the area measurement (using the reference model) is equivalent to the uncertainty associated with the area estimate of the reference standard itself. Furthermore, if the permissible error of the AVMS is designated as ΔA, and a target error ratio of 3:1 is desired, then the standard's error is $\Delta A/3$. The known relative error can be used to estimate the standard uncertainty, assuming a rectangular distribution of the measurement error. Under these assumptions, the calibration uncertainty of the reference standard can be calculated using the following equation:

$$u(l) = \frac{\delta A \cdot a \cdot b}{100\% \cdot 3 \cdot \sqrt{3} \cdot (a + b)}, \text{m} \quad \text{(A3)}$$

A rectangular distribution is also assumed when calculating the expanded uncertainty. $U(l) = \sqrt{3} \cdot u(l)$. Based on the above information, acceptance criteria for the reference instrument's length measurements can be determined, as given by the (eqn. A4):

$$U(l) \leq \frac{\delta A \cdot a \cdot b}{100\% \cdot 3 \cdot (a + b)}, \text{m} \quad \text{(A4)}$$

The standard uncertainty of the length measurement for reference instruments (such as tape measures, rulers, or laser distance meters) is directly related to the uncertainty of the area assessment performed with that standard and to the dimensions of the area being measured. If we consider the area as a rectangle, the lengths of its sides (see Fig. 1) will influence how the standard's length calibration uncertainty (in absolute terms) depends on the size of the area. Fig. 1 illustrates this relationship.

For illustration, consider rectangular areas A of 2 m², 5 m², 10 m², 15 m², 20 m², 25 m², and 30 m². These could correspond to rectangles with side lengths (a×b) of approximately (1×2) m, (1×5) m, (2×5) m, (2×7.5) m, (2×10) m, (2×12.5) m, and (3×10) m, respectively. Assuming a rectangular distribution of area measurement error, the standard uncertainty $u(\Delta A) = \frac{\Delta A}{\sqrt{3}}$ for these example areas would be about 0.02 m², 0.05 m², 0.1 m², 0.15 m², 0.2 m², 0.25 m², and 0.3 m², respectively (these values correspond to the maximum allowable errors for those area sizes under a certain error criterion). In practice, to measure the side lengths of a model that produces areas in this range, one should use length standards with an expanded calibration uncertainty on the order of 7–24 mm (appropriate for the typical lengths involved). This implies that standard measuring tools such as tapes, rulers, or laser meters with a scale division of 0.5–1 mm are sufficient for the task.