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Supplementary Material

Tab. S1 - The taper functions of four models evaluated based on fitting data.

Model	Expression
Taper	$d = \left\{ \begin{array}{c} I_S\left[D^2\left(1+\frac{\left(1-\frac{h}{H}\right)^{b_1}-\left(1-\frac{1.30}{H}\right)^{b_1}}{1-\left(1-\frac{1.30}{H}\right)^{b_1}}\right)\right] \\ +I_B\left[D^2-\frac{(D^2-F^2)\left(\left(1-\frac{1.30}{H}\right)^{b_2}-\left(1-\frac{h}{H}\right)^{b_2}\right)}{\left(1-\frac{1.30}{H}\right)^{b_2}-\left(1-\frac{5.30}{H}\right)^{b_2}}\right] \\ +I_T\left[F^2\left(b_4\left(\frac{h-5.30}{H-5.30}-1\right)^2+I_M\left(\frac{1-b_4}{b_3^2}\right)\left(b_3-\frac{h-5.30}{H-5.30}\right)^2\right)\right] \end{array} \right\}^{0.5}$ $I_S = 1.if\ \ h < 1.3; 0\ otherwise$ $I_B = 1.if\ \ 1.3 \le h \le 5.3; 0\ otherwise$ $I_T = 1.if\ \ h > 5.3; 0\ otherwise$ $I_M = 1.if\ \ h < \left(5.30 + b_3(H-5.30)\right); 0\ otherwise$
(Model 1) Jiang et al. (2005)	Volume $V = k\left[\begin{array}{l} I_1 D^2\left[(1-GW_1)(U_1-L_1)+\frac{W_1\left(\left(1-\frac{L_1}{H}\right)^{b_1}(H-L_1)-\left(1-\frac{U_1}{H}\right)^{b_1}(H-U_1)\right)}{(b_1+1)}\right] \\ +I_2 I_3\left[T(U_2-L_2)-\frac{Z\left(\left(1-\frac{L_2}{H}\right)^{b_2}(H-L_2)-\left(1-\frac{U_2}{H}\right)^{b_2}(H-U_2)\right)}{(b_2+1)}\right] \\ +I_4 F^2\left[\begin{array}{l} b_4(U_3-L_3)-\frac{b_4((U_3-5{,}30)^2-(L_3-5{,}30)^2)}{(H-5{,}30)}+ \\ \frac{\left(\frac{b_4}{3}\right)((U_3-5{,}30)^3-(L_3-5{,}30)^3)}{(H-5{,}30)^2}+ \\ \frac{I_5\left(\frac{1}{3}\right)\left(\frac{(1-b_4)}{b_3^2}\right)\left(b_3(H-5{,}30)-(L_3-5{,}30)\right)^3}{(H-5{,}30)^2}- \\ \frac{I_6(1/3)\left((1-b_2)/b_4^2\right)\left(b_4(H-5{,}30)-(U_3-5{,}30)\right)^3}{(H-5{,}30)^2} \end{array}\right] \end{array}\right]$ $G = \left(1-\frac{1{,}30}{H}\right)^{b_1} \quad W_1 = \frac{1}{1-G} \quad X = (1-1{,}30/H)^{b_2} \quad Y = \left(1-\frac{5{,}30}{H}\right)^2$ $T = D^2 ZX \quad Z = \frac{D^2-F^2}{X-Y} \quad L_1 = max(L, 0.30) \quad L_2 = max(L, 1.30)$ $L_3 = max(L, 5.30) \quad U_1 = min(L, 1.30) \quad U_2 = min(L, 5.30) \quad U_3 = min(L, H)$ $I_1 = \begin{cases} 1 & L < 1{,}30 \\ 0 & other \end{cases} \quad I_2 = \begin{cases} 1 & L < 5{,}30 \\ 0 & other \end{cases} \quad I_3 = \begin{cases} 1 & U > 1{,}30 \\ 0 & other \end{cases} \quad I_4 = \begin{cases} 1 & U > 5{,}30 \\ 0 & other \end{cases}$ $I_5 = \begin{cases} 1 & (L_3-5{,}30) < b_3(H-5{,}30) \\ 0 & otherwise \end{cases} \quad I_6 = \begin{cases} 1 & (U_3-5{,}30) < b_3(H-5{,}30) \\ 0 & otherwise \end{cases}$

Tab. S1 – (continuation).

Model		Expression
(Model 2) Max & Burkhart (1976)	Taper	$d = D[b_1(Z-1) + b_2((Z)^2-1) + b_3(a_1 - Z)^2 I_1 + b_4(a_2 - Z)^2 I_2]^{0.5}$ $I_1 = 1.if\ \ Z \leq\ a_1; 0\ otherwise$ $I_2 = 1.if\ \ Z \leq\ a_2; 0\ otherwise$ $Z = \frac{h}{H}$
	Volume	$V = kD^2H\left(\frac{b_1}{2} + \frac{b_2}{3} - (b_1 + b_2) + \frac{b_3}{3}a_1^3 + \frac{b_4}{3}a_2^3\right)$
(Model 3) Parresol et al. (1987)	Taper	$d = D\{Z_i^2(b_1 - b_2 z_1) + (z_i - a)^2[b_3 + b_4(z_i + 2a)]I\}^{0.5}$ $I = 1.if\ \ Z \geq\ a; 0\ otherwise$ $Z = \frac{H-h}{H}$
	Volume	$V = kD^2H\left(\frac{b_1}{3} + \frac{b_2}{4} + \frac{a_1^3 b_3}{3} + \frac{3a_1^4 b_4}{4}\right)$
(Model 4) Fang et al. (2000)	Taper	$d = c_1\left[H^{(k-b_1)/b_1}(1-Z)^{(k-\beta)/\beta}\alpha_1^{I_1+I_2}\alpha_2^{I_2}\right]^{0.5}$ $I_1 = 1.if\ \ p_1 \leq q \leq\ p_2; 0\ otherwise$ $I_2 = 1.if\ \ p_2 \leq q \leq\ 1; 0\ otherwise$ $c_1 = \sqrt{\frac{a_1 D^{a_2} H^{a_3 - k/b_1}}{b_1(r_0 - r_1) + b_2(r_1 - \alpha_1 r_2) + b_3(\alpha_1 r_2)}}$ $\beta = b_1^{1-(I_1+I_2)} b_2^{I_1} b_3^{I_2}$ $\quad \alpha_1 = (1-p_1)^{(b_2-b_1)k/b_1 b_2}$ $\quad \alpha_2 = (1-p_2)^{(b_3-b_2)k/b_2 b_3}$ $r_0 = \left((1-h_{st})/H\right)^{k/b_1}$ $\quad r_1 = (1-p_1)^{k/b_1}$ $\quad r_2 = (1-p_2)^{k/b_2}$ $p_1 = h_1/H$ $\quad p_2 = h_2/H$ $\quad Z = h/H$
	Volume	$V = a_1 D^{a_2} H^{a_3}$

where d is diameter outside bark at a height h (cm), D is diameter outside bark at breast height (cm), h is height above ground to the measurement point (m), H is total tree height (m), a_1 and a_2 are the join points to be estimated from the sample data for Max & Burkhart (1976) model, a is the join point to be estimated from the sample data for Parresol et al. (1987) model, F diameter outside bark at 5.3 m above ground (cm), h_1 and h_2 are the heights from ground level where the two inflection points assumed in the Fang et al. (2000) model occur, k is equal to π/40 000, a metric constant for converting from diameter squared in square centimeters to cross-sectional area in square meters, h_{st} is stump height (m), a_1-a_3, b_1-b_4, p_1- p_2 are coefficients to be estimated.

Fig. S1 - The location of the case study area.