Hirigoyen A, Navarro-Cerrillo R, Bagnara M, Franco J, Resquin F, Rachid-Casnati C (2021).
Modelling taper and stem volume considering stand density in *Eucalyptus grandis* and *Eucalyptus dunnii*
iForest – Biogeosciences and Forestry – doi: 10.3832/ifor3604-014

Supplementary Material

Tab. S1 - Compatible taper and volume systems tested for *Eucalyptus grandis* and *Eucalyptus dunnii* plantations in Uruguay.

Reference	Systems
(Kozak et al. 1969)	Taper function: $d_i^2 = D^2\left[b_1(q-1) + b_2(q^2-1)\right]$ Volume equation: $V = \beta D^2 H$ Compatibility relationship (Martin 1981): $\beta = -k\left(\frac{b_1}{2} + \frac{2b_2}{2}\right)$
(Max & Burkhart 1976)	Taper function: $d_i^2 = D^2\left(b_1(q-1) + b_2(q^2-1) + b_3(a_1-q)^2 I_1 + b_4(a_2-q)^2 I_2\right)$ where $\begin{cases} I_1 = 1 \text{ if } q \le a_1;\ 0 \text{ otherwise} \\ I_2 = 1 \text{ if } q \le a_2;\ 0 \text{ otherwise} \end{cases}$ Volume equation: $V = \beta D^2 H$ Compatibility relationship (Martin 1981): $\beta = k\left(\frac{b_1}{2} + \frac{b_2}{3} - (b_1 + b_2) + \frac{b_3}{3}a_1^3 + \frac{b_4}{3}a_2^3\right)$
(Fang et al. 2000)	Taper function: $d_i = c_1 \sqrt{H^{(k-b_1)/b_1}(1-q)^{(k-\beta)/\beta}\alpha_1^{I_{1+I_2}}\alpha_2^{I_2}}$ where $\begin{cases} I_1 = 1 \text{ if } p_1 \le q \le p_2;\ 0 \text{ otherwise} \\ I_2 = 1 \text{ if } p_2 \le q \le 1;\ 0 \text{ otherwise} \end{cases}$ p_1 and p_2 are relative heights from ground level where the two inflection points are assumed by the model to occur, dividing the stem in three sections. $\beta = b_1^{1-(I_1+I_2)} b_2^{I_1} b_3^{I_2} \alpha_1 = (1-p_1)^{\frac{(b_2-b_1)k}{b_1 b_2}};\ \alpha_2 = (1-p_2)^{\frac{(b_3-b_2)k}{b_2 b_3}};\ r_0 = \left(1-\frac{h_{st}}{H}\right)^{k/b_1};\ r_1 = (1-p_1)^{k/b_1};$ $r_2 = (1-p_2)^{k/b_2}$ $c_1 = \sqrt{\frac{a_0 D^{a_1} H^{a_{2-k/b_1}}}{b_1(r_0-r_1) + b_2(r_1-\alpha_1 r_2) + b_3\alpha_1 r_2}}$ Merchantable volume equation: $v = c_1^2 H^{k/b_1}\left[b_1 r_0 + (I_1+I_2)(b_2-b_1) r_1 + I_2(b_3-b_2)\alpha_1 r_2 - \beta(1-q)^{k/\beta}\alpha_1^{I_1+I_2}\alpha_2^{I_2}\right]$ Volume equation: $V = a_0 D^{a_1} H^{a_2}$

Reference	Systems
(Sharma & Oderwald 2001)	Taper function: $d_i^2 = D^2\left(\frac{h_i}{1.3}\right)^{2-a_1}\left(\frac{H-h_i}{H-1.3}\right)$ Volume equation: $V = a_0 D^{a_1} H^{3-a_1}$ Compatibility relationship: $a_0 = \frac{k\,(D/1.3)^{2-a_1}}{\left(1-\frac{1.3}{H}\right)(3-a_1)(4-a_1)}$

where k is π/40,000, a metric constant; b_i is the form factor of tree section i; D is the diameter at breast height over bark (1.3 m above ground, cm); d_i (cm) is the diameter over bark at height h_i (m); H is the total tree height (m); q is h_i/H; h_{st} is the stump height (m) and a_0, a_1, a_2, b_1, b_2, b_3, p_1 and p_2 are parameters to be estimated.